6th International Conference on Earthquake Geotechnical Engineering
1-4 November 2015
Christchurch, New Zealand

A Double Hyperbolic Model

N. Yoshida[1]

ABSTRACT

A new stress-strain model, named "double hyperbolic model (DHP model)" is proposed. This model is composed of two hyperbolic models. The first hyperbolic model is used at strains less than the reference strain (strains at stiffness degradation ratio is 0.5) in which the reference strain is used as a parameter, and the second hyperbolic model is used at strains larger than the reference strain, in which shear strength is used as a parameter. Two models are connected at the reference strain. It uses only two parameters and can simulate behavior in wide range of strains from very small to large strains. Accuracy and applicability of the model is examined by using about 500 cyclic shear deformation characteristics test results. Conventional models, the hyperbolic model and the Ramberg-Osgood model, are also examined by the same method. It is found that the error of the double hyperbolic model is much smaller than the conventional models.

Introduction

An Engineer must choose stress-strain models for the seismic response analysis of ground depending on the soil data he has. If he conducts cyclic shear deformation characteristics tests to obtain the strain dependent shear modulus and damping ratio (cyclic shear deformation characteristics in the following), the stress-strain model proposed by the authors (Ishihara et al, 1985; Yoshida et al., 1990) gives perfect simulation. On the other hand, if he does not conduct cyclic shear deformation characteristics test, stress-strain parameters are estimated from such parameters as soil type and SPT-*N* value. In these cases, stress-strain models that have a few parameters such as the hyperbolic and the Ramberg-Osgood models are preferable.

The authors collected about 500 sets of data on cyclic shear deformation characteristics and used them to examine the applicability of these stress-strain models (Yoshida and Wakamatsu, 2012 and 2013). It was concluded that the hyperbolic model has a tendency to underestimate shear stress and the Ramberg-Osgood model has a tendency to overestimate shear stress at large strains as will be shown later. Therefore applicability of these models is limited at large strains. A new stress-strain model is proposed in this paper, which can simulate soil behavior over a wide range of strains up to the shear strength.

Brief Review of the Previous Research

The authors collected about 500 data sets on cyclic shear deformation characteristics test results, which are classified based on the geologic age, depositional environment and soil type as shown in Table 1. They are collected from 95 sites in the Tokyo Metropolitan Area (Kanto district),

[1] Professor, Department of Civil and Environmental Engineering, Tohoku Gakuin University, Tagajo, Japan, yoshidan@mail.tohoku-gakuin.ac.jp

Japan. Test methods are cyclic triaxial test, cyclic torsion test combined with resonant test using circular solid specimen, and cyclic hollow cylinder torsion test. Initial confining stress σ'_m of each test is shown in Table 1 and Figure 1 (the legend in Figure 1 is used in the following figures). The cyclic shear deformation characteristics of all soils are sown in Figure 2. Here, G denotes secant shear modulus, G_0 denotes initial shear modulus, γ denotes shear strain, and h denotes damping ratio. In the figure, red dashed line indicates clayey soil and blue solid line indicates sandy soil. Although measured shear strains are different in each test, G/G_0 and h are interpolated at strains 1, 2 and 5 in each digit by using a Bezier curve. Recently, large strain behavior is required because design earthquake motion becomes large. On the other hand, applicability of the conventional cyclic shear deformation characteristics test is supposed to be a little larger than 0.1 % (JGS Committee, 2013), but, modulus and damping ratio are frequently measured at strains larger than1 %. Considering these situation, strains from 10^{-6} to 0.01 are used in this study. Therefore there are 13 G/G_0-γ and h-γ data points in each test data.

Table 1. Classification of cyclic shear deformation characteristics test data

Geologic age		Depositional environment	Soil Type	Geologic category	Number of data			
					σ'_m	I_p	F_c	D_{50}
Man-made		Fill	Clayey soil	1-Bc	10	7	10	10
			Sandy soil	2-Bs	14	2	14	14
Holocene	Upper	Aeolian	Sandy soil	3-As	2	0	0	0
		Marine	Sandy soil	4-As	13	0	13	13
			Gravel	5-Ag	0	0	0	0
		Brackish-water	Clayey soil	6-Ac	12	10	12	12
			Sandy soil	7-As	15	3	15	15
		Fluvial	Peat	8-Ap	3	2	2	2
			Clayey soil	9-Ac	30	29	30	25
			Sandy soil	10-As	28	0	28	28
			Gravel	11-Ag	0	0	0	0
	Lower	Marine	Clayey soil	12-Ac	119	99	113	113
			Sandy soil	13-As	20	3	18	18
		Fluvial	Gravel	14-Ag	0	0	0	0
Pleistocene	Upper	Marine Brackish-water	Clayey soil	15-Ac	17	15	17	17
			Sandy soil	16-As	5	1	5	5
	Upper	Volcanic ash fall	Loam	17-Lm	15	14	14	14
			Clayey soil	18-Dc	6	5	6	6
			Sandy soil	19-Ds	0	0	0	0
			Gravel	20-Dg	0	0	0	0
	Middle	Volcanic ash fall	Loam	21-Lm	0	0	0	0
			Clayey soil	22-Dc	76	53	71	70
			Sandy soil	23-Ds	92	1	74	74
			Gravel	24-Dg	1	0	1	1
Pliocene-Pleistocene	Lower Pleistocene Upper Pliocene	Marine	Mudstone	25-Dc	4	4	4	4
			Sandy-gravel rock	26-Dsg	0	0	0	0

The applicability of the frequently used the hyperbolic model the Ramberg-Osgood mode (R-O model):

$$\text{Hyperbolic model: } \tau = \frac{G_0 \gamma}{1+\gamma/\gamma_r} \tag{1}$$

$$\text{Ramberg-Osgood model: } \gamma = \frac{\tau}{G_0}\left\{1+\alpha\left(\frac{\tau}{\tau_f}\right)^{\beta-1}\right\} \tag{2}$$

is examined in Figures 3 and 4. Here, τ denotes shear stress, γ_r denotes reference strain τ_f denotes shear strength, and α and β are parameters. The hyperbolic model uses one parameter and the Ramberg-Osgood model uses two independent parameters to express nonlinear characteristics in addition to the elastic modulus G_0.

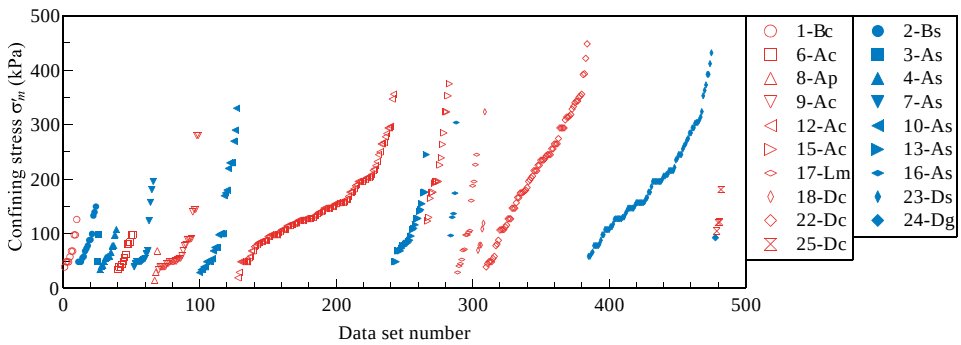


Figure 1. Distribution of initial effective mean stress σ'_m

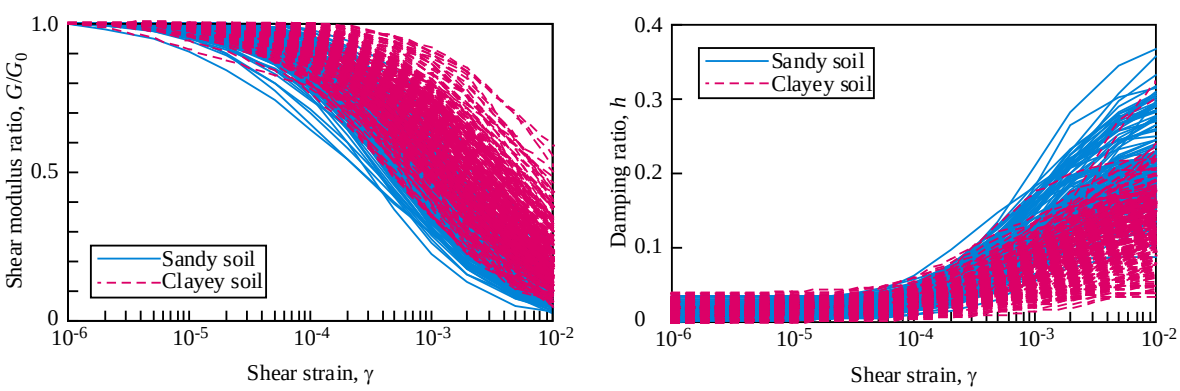


Figure 2. Cyclic shear deformation characteristics of all test specimens

The reference strain in the hyperbolic model is evaluated at a strain where G/G_0=0.5 in Figure 3. Agreement at small strain is good, but shear stress is underestimated at large strains in almost all data. Two parameters of the Ramberg-Osgood model are evaluated so that test and model agrees at G/G_0=0.5 and 0.8 in order to get good agreement at relatively small strains in Figure 4; shear stresses at large strains is overestimated in this model. From these observations, large strain behavior is difficult to simulate by the conventional stress-strain models although

behavior at small strain simulated well. In our previous study (Yoshida and Wakamatsu, 2012), several methods are examined to calculate values of parameters and evaluate disagreement or error of these models. Here error of i-th test data E_i is calculated by

$$E_i = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\left(\frac{\tau}{G_0}\right)_{k,\mathrm{test}} - \left(\frac{\tau}{G_0}\right)_{k,\mathrm{model}}\right)^2} = \frac{1}{G_0}\sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\tau_{k,\mathrm{test}} - \tau_{k,\mathrm{model}}\right)^2} \tag{3}$$

where N denotes number of data and is 13 as explained before, and subscripts "test" and "model" denote test data and model. Error can be smaller than the cases shown in Figures 3 and 4 if parameters another set of parameters, but general tendency is the same.

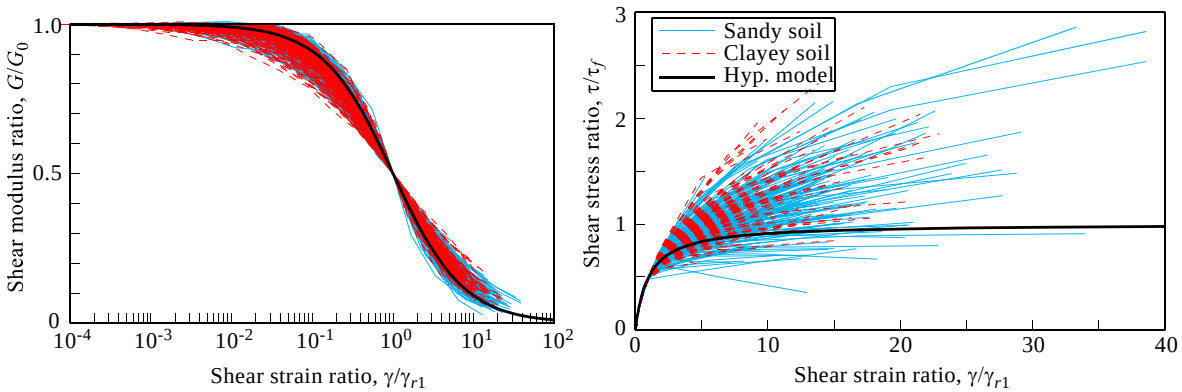


Figure 3. Applicability of hyperbolic model

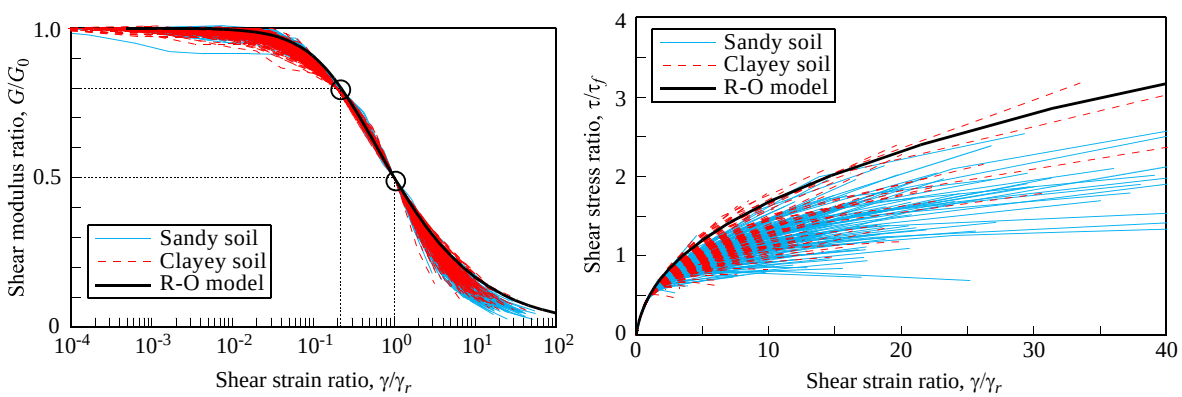


Figure 4 Applicability of Ramberg-Osgood model

A Double Hyperbolic Model

The model proposed in this paper is composed of two hyperbolic models, and is named a double hyperbolic model (DHP model). Two equations are

$$\tau = \frac{G_0 \gamma}{1 + \gamma / \gamma_r} \qquad (\gamma \le \gamma_r) \tag{4}$$

$$\tau = \frac{G_0 (\gamma - \gamma_0)}{A + B(\gamma - \gamma_0)} \qquad (\gamma > \gamma_r) \tag{5}$$

where A, B and γ_0 are parameters. These parameters are determined under the following conditions.

1) Two models have the same values at $\gamma = \gamma_r$ and the slope at $\gamma = \gamma_r$ is continuous.
2) Shear strength is τ_f.

Then Eq. (5) yields

$$\tau = G_0 \gamma_r \frac{k\gamma / \gamma_r + k - 1}{4k - 3 + \gamma / \gamma_r} \qquad (\gamma > \gamma_r) \tag{6}$$

Here, the parameter $k = \tau_f / (\gamma_r G_0)$ is called shear strength ratio in the following because $G_0 \gamma_r$ is the shear strength parameter of the first (small strain) hyperbolic model. Shear strength ratio takes the value larger than 0.5. The conventional hyperbolic model is obtained by setting k=1. Figure 5 shows change of the cyclic shear deformation characteristics depending on k value.

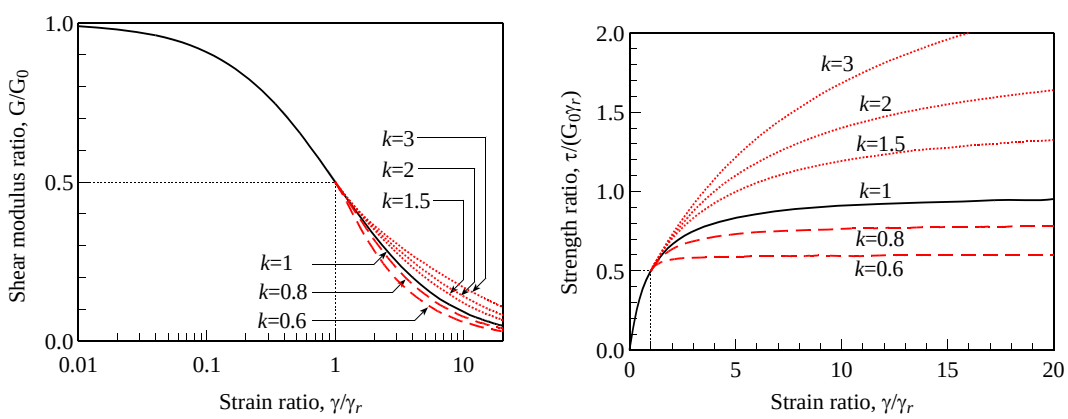


Figure 5. Parametric study of double hyperbolic model

Parameters and Accuracy

Among the two parameters used in the double hyperbolic, the reference strain is already studied in detail in our previous study (Yoshida and Wakamatsu, 2013). Therefore, values of shear strength ratio *k* are studied in this paper. They are evaluated by two different methods, i.e., least square method and the nonlinear method.

Equation (6) is re-written as

$$\gamma_r + \frac{\tau}{G_0}\left(\frac{\gamma}{\gamma_r} - 3\right) = k\left(\gamma_r + \gamma - 4\frac{\tau}{G_0}\right) \tag{7}$$

This equation indicates that relationships between "$\gamma_r + \tau / G_0 \left(\gamma / \gamma_r - 3\right)$" and "$\gamma_r + \gamma - 4\tau / G_0$" are linear. Then the shear strength ratio is obtained by the least square method. Strains larger than the reference strain is used to evaluate shear strength ratio.

An iterative procedure is used to make the error in Eq. (3) minimum by changing *k* in the second method, which is called a nonlinear method in the following because this process is equivalent to solving a nonlinear equation. In our previous study on the hyperbolic model, reference strains obtained both by the least square method and by the nonlinear method are almost identical. Therefore, in the nonlinear method, all the data is used to evaluate the error because the shear strength ratio obtained is expected to be same as the one by the least square method if strains larger than the reference strain are used.

Shear strength ratios obtained by two methods are shown in Figure 6. Values of *k* scatter widely up to 9. Many of them are larger than 1.0, which agrees with the fact that hyperbolic model shown in Figure 3 underestimates shear stress at large strains.

Shape of distributions in Figure 6 is similar, which can also be confirmed from the comparison of shear strength ratio in Figure 7. Many points lie on the 1:1 line. In the same manner, Errors by two methods are compared in Figure 8, which also lie on the 1:1 line although errors by the nonlinear methods is a little smaller than those by the least square method. These observations indicate that agreement at large strains is important to make the error small.

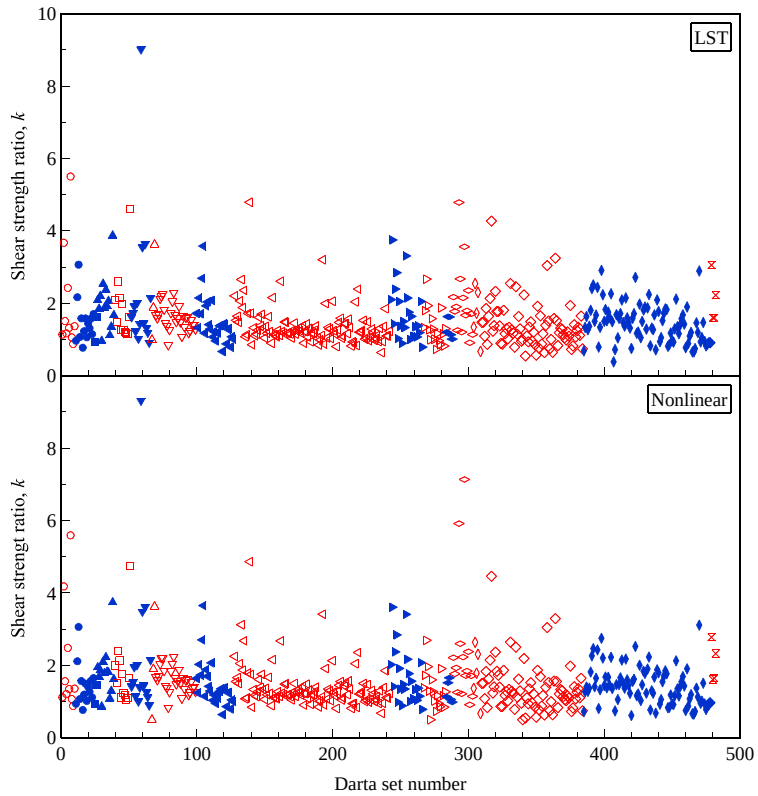


Figure 6 Shear strength ratios obtained by least square and nonlinear methods

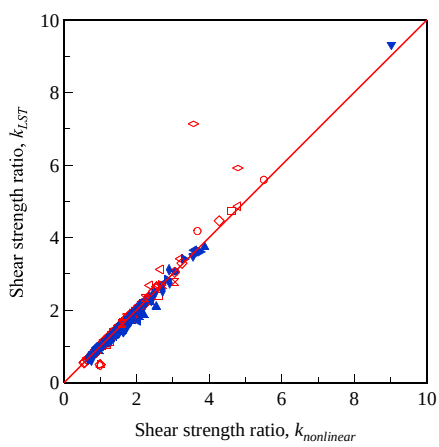


Figure 7 Comparison of shear strength ratio

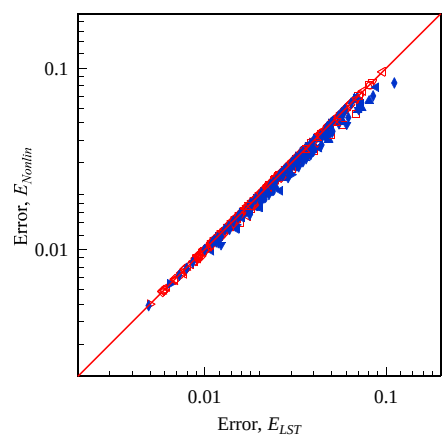


Figure 8 Comparison of error

Figure 9 shows errors E_i of all data for three models. Here model parameters are evaluated to make the error minimum by using the iterative nonlinear methods in all models. Generally speaking, errors on the Ramberg-Osgood model is the largest and those by the DHP model is the smallest. In order to see accuracy in detail, error by the DHP model is compared with other models in Figure 10.

Maximum error of the DHP model is about 0.1, and that of the hyperbolic model is about 0.2. Average error by the hyperbolic model is several times larger than that of DHP mode. On the other hand, that of the Ramberg-Osgood model is much larger up to 1.05.

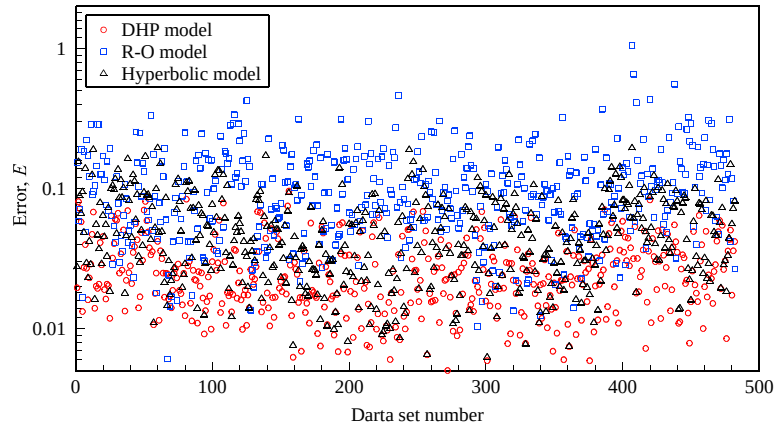
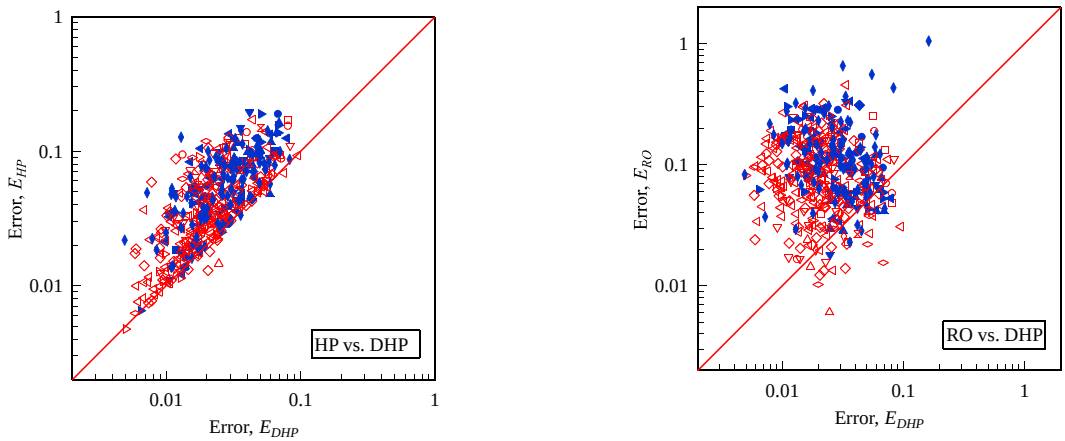


Figure 9 Minimized error of each model



(a) Hyperbolic model (b) Ramberg-Osgood model

Figure 10 Comparison of errors with other models

Concluding Remarks

A new stress-strain model, named a double hyperbolic (DHP) model, is proposed. This model uses only two parameters in expressing the nonlinear behavior, which are the reference strain (strain at which G/G_0=0.5) and the shear strength (shear stress ratio is calculated from the shear modulus and the reference strain). Simulation of about 500 cyclic shear deformation characteristics shows that error of this model is much smaller than those by the conventional stress-strain models (hyperbolic model and Ramberg-Osgood model). The mechanical meaning of the parameters is very clear, which is another advantage of this model.

Here, it is noted that shear strength use in evaluating k is not the shear strength obtained by a

monotonic loading test nor calculated from the Mohr-Coulomb criteria. Figure 11 show test results of monotonic and cyclic loading test results; result of the cyclic test is a part of the cyclic shear deformation characteristics test and monotonic test is carried under drained and undrained conditions (Mikami et al., 2006). Stresses at large strains by the cyclic loading test are much smaller than those by the monotonic loading test because of the excess porewater pressure generation. Therefore the relevant evaluation of the shear strength for the DHP model remains a problem for the future study.

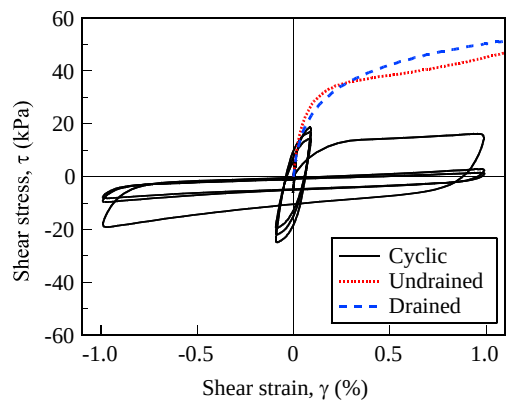


Figure 11 Stress-strain curves under static and cyclic loading

This paper deals with only the skeleton or backbone curve, and hysteresis curve or damping characteristics are not discussed. They are already discussed in our previous study (Yoshida and Wakamatsu, 2012), in which empirical equation to obtain the maximum damping ratio h_{max} for the Hardin-Drnevich model (Hardin and Drnevich, 1972)

$$h = h_{max}(1 - G/G_0) \tag{8}$$

is shown. The hysteresis curve can be calculated by using the method proposed by Ishihara et al. (1985).

References

Hardin, B. O. and Drnevich, V. P., *Shear modulus and damping in soils: design equations and curves*, Proc. of the ASCE, 98 (SM7), 667-692, 1972

Ishihara, K., Yoshida, N, Tsujino, S. *Modelling of stress-strain relations of soils in cyclic loading*, 5 ICNMG, Nagoya, **1**, 373-380, 1985

JGS Committee on cyclic shear deformation characteristics considering large strains, Proc. *Symposium on cyclic shear deformation characteristics considering large strains*, JGS, 180pp., 2013 (in Japanese)

Mikami, T., Yoshida, N. and Sawada, S., *Laboratory test on cyclic behavior of soil for earthquake response analysis* (*Part 2*), 41st Japan National Conference on Geotechnical Engineering, 417-418, 2006 (in Japanese)

Yoshida, N., Tsujino, S. and Ishihara, K. *Stress-strain model for nonlinear analysis of horizontally layered deposit*, Summaries of the Technical Papers of Annual Meeting of AIJ, Chugoku, **B**, 1639-1640, 1990 (in Japanese)

Yoshida, N. and Wakamatsu, K. *Modeling of cyclic deformation characteristics considering geological age and*

depositional conditions, 2nd International Conference on Performance-based Design in Earthquake Geotechnical Engineering, Taormina, 455-466, 2012

Yoshida, N. and Wakamatsu, K. *Modeling of cyclic shear deformation characteristics considering geologic age and depositional environment*, Japanese Geotechnical Journal, **8 (2)**, 265-284, 2013 (in Japanese)