6th International Conference on Earthquake Geotechnical Engineering
1-4 November 2015
Christchurch. New Zealand

Numerical Modelling of Multi-directional Earthquake Loading and Its Effect on Sand Liquefaction

V. Tsaparli[1], S. Kontoe[2], D.M.G. Taborda[3], D.M. Potts[4]

ABSTRACT

Earthquakes generate multi-directional ground motions, two components in the horizontal direction and one in the vertical. Nevertheless, the effect of vertical motion on site response analysis has not been the object of extensive research. The 2010/2011 Canterbury sequence of seismic events in New Zealand is a prime example among other earlier field observations strongly corroborating that the vertical acceleration may have a detrimental effect on soil liquefaction. Consequently, this study aims to provide insight into the influence of the input vertical motion on sand liquefaction. For this reason, two ground motions, with very different frequency contents, are used as the input excitations. Non-linear elasto-plastic plane strain fully coupled effective stress-based finite element analyses are conducted to investigate the occurrence of liquefaction in a hypothetical fully saturated Fraser River Sand deposit. The results indicate that the frequency content of the input motion is of utmost importance for the response of sands to liquefaction when the vertical loading is considered.

Introduction

Current design guidelines focus predominantly on the implications of the horizontal component of the ground motion only, with the vertical acceleration drawing very limited attention. The latter is conventionally included in design by scaling the horizontal design spectrum, considering that the spectral ratio of peak vertical to peak horizontal acceleration, V_{peak}/H_{peak}, will not exceed a value of 2/3 (International Code Council, 1996). The main argument is that the vertical component is traditionally considered to be of much lower amplitude than the horizontal one. Additionally, it is thought not to affect the liquefaction potential of sandy deposits; P-waves produce only compressive stresses that are transmitted through the soil's pore water and hence affect the total stresses only and not the effective stress state (Ishihara, 1996).

However, recent as well as earlier field observations indicate that the vertical acceleration can have a detrimental effect on soil liquefaction and should be included in ground response analyses. Unexpectedly high vertical ground accelerations have been recorded in past earthquake events: ratios of V_{peak}/H_{peak} in excess of the conventional value of 2/3 being observed in both the seismic events of Northridge, California, 1994 and Kobe, Japan, 1995. More recently, the Canterbury earthquake sequence of the 2010 and 2011 seismic events in New Zealand strongly corroborates the fact that there may be a relation between high vertical components of acceleration and soil liquefaction.

[1]Miss Vasiliki Tsaparli, Dep. of Civil Eng., Imperial College, London, UK, vasiliki.tsaparli10@imperial.ac.uk
[2]Dr. Stavroula Kontoe, Dep. of Civil Eng., Imperial College, London, UK, stavroula.kontoe@imperial.ac.uk
[3]Dr. David M.G. Taborda, Dep. of Civil Eng., Imperial College, London, UK, d.taborda@imperial.ac.uk
[4]Prof. David M. Potts, Dep. of Civil Eng., Imperial College, London, UK, d.potts@imperial.ac.uk

Over the years there have been a limited number of studies that have explored the effect of multi-directional seismic loading on the liquefaction response of fully saturated sand deposits (Shiomi & Yoshizawa, 1996; Yang et al., 2002), all concluding that the liquefaction response is not affected by the vertical loading. The aim of the current study is to investigate further this phenomenon by focusing on the particular nature and frequency content of the input excitation.

Modelled Sand Deposit and Input Ground Motions

A hypothetical soil deposit consisting of Fraser River Sand (FRS) with a thickness of 32 m, a relative density of 40% and a permeability of 4.2E-04 m/s was considered in the performed finite element analyses. Fraser River Sand is a material of alluvial origin deposited in the Fraser River Delta in Vancouver, British Columbia (Williams, 2014). The sand deposit was assumed to be fully saturated with the water table specified at ground level, underlain by impermeable rigid bedrock. The deposit is shown in Figure 1. As the aim of the study is to investigate the influence of the input motion on the liquefaction response of sands, the depth of the deposit was chosen such that it maximizes this effect. The shear wave velocity in the deposit increases with depth following a non-linear distribution and has an average value of 160 m/s, whereas the respective value of the compressional wave velocity is mainly controlled by the bulk stiffness of the water ($2.2x10^6$ kPa) and is equal to 1609 m/s. Given the above, the average fundamental frequency of the FRS deposit considering S-waves is 1.26Hz, whereas the corresponding frequency for P-waves is 12.67 Hz.

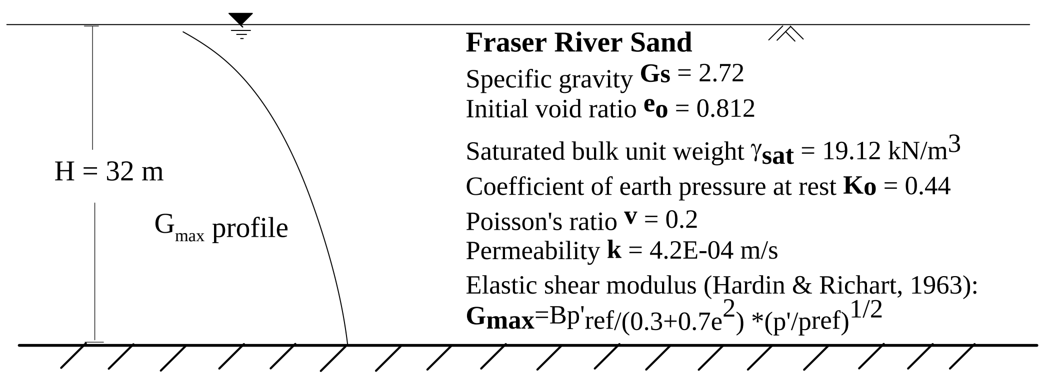

Figure 1. Sand deposit used in the FE analyses

Two ground motions were used in this study. The first one is the outcrop motion from the Christchurch, New Zealand, seismic event of 22nd February 2011, with a magnitude of Mw = 6.2. This was recorded in the Lyttelton Port Company (LPCC) strong ground motion station, about 10 km south-east of Christchurch, in an area underlain by a volcanic rock outcrop. This event was characterized by the greatest intensity amongst the events of the Canterbury earthquake series and showed the most detrimental effects in terms of soil liquefaction. The second motion used is the one recorded at 47 m depth in a downhole array from the 20th May 1986 Lotung, Taiwan, seismic event, with an estimated local magnitude of $M_{L=}$ 6.5 (Elgamal et al., 1995). The acceleration time-histories of the horizontal and vertical components of the two ground motions are shown in Figure 2. The respective Fourier Spectra are also included in the same figure. The peak horizontal (PHA) and peak vertical (PVA) surface accelerations of the Christchurch event had a value of 0.87g and 0.4g, respectively, resulting in a spectral ratio, V_{peak}/H_{peak}, of 0.45, while the corresponding values for the Lotung motion were 0.1g and 0.03g, resulting in a spectral ratio of 0.33. However, it should be noted that for this study both components of the ground motion of the Lotung event were scaled up, so that its PHA and PVA matched those of the Christchurch event.

The selection of the input motions for the numerical analyses was such that their frequency content is profoundly different; as it can be seen in Figure 2, although both components of the scaled up Lotung event are characterized by larger values of Fourier amplitude, the frequency content of the two components of the Christchurch event is almost three times wider, with significant components up to about 30 Hz. This difference can also be clearly seen from the values of the mean periods, T_m, of the two events: 0.23 s for the Christchurch event, compared to 0.93 s of the Lotung event. The value of T_m was based on Equation 1 by Rathje et al. (2004):

$$T_m = \frac{\sum_i C_i^2 (\frac{1}{f_i})}{\sum_i C_i^2} \quad \text{for } 0.25 \text{ Hz} \leq f_i \leq 20 \text{ Hz and } \Delta f \leq 0.05 \text{ Hz}, \tag{1}$$

where C_i are the coefficients of the Fourier amplitude and fi and Δf are the frequencies and the frequency step, respectively, in the discrete fast Fourier transform.

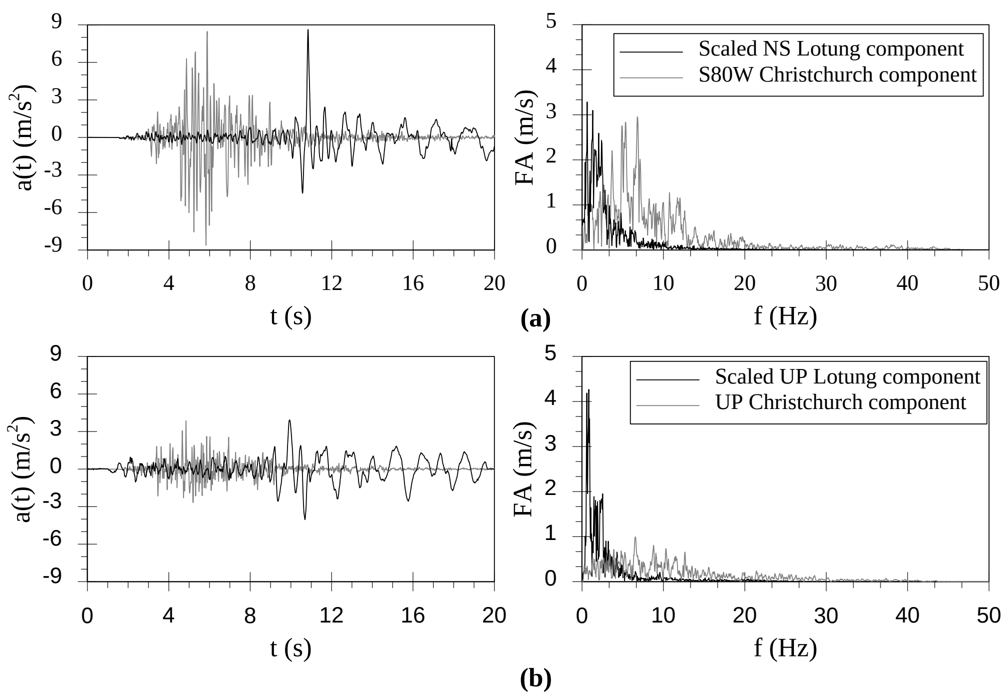

Figure 2. Acceleration time-histories and Fourier Spectra of input ground motions – (a) horizontal components and (b) vertical components

Numerical Method and Constitutive Model

Six nonlinear elasto-plastic effective stress-based fully coupled plane strain finite element analyses were carried out with the Imperial College Finite Element Program – ICFEP (Potts &

Zdravković, 1999). The mesh generated consists of a column of 127x1 8-noded quadrilateral elements with dimensions of 0.25x0.25 m^2. These were chosen based on the recommendations by Bathe (1996) for 8-noded solid element, in order to guarantee that the mesh is sufficiently fine so as not to filter out waves characterised by short wavelengths. In terms of boundary conditions, tied degrees of freedom were used along the vertical boundaries of the mesh during the dynamic analysis (Zienkiewicz et al., 1988), whereas vertical or horizontal displacements were prescribed to be zero along the bottom boundary when the horizontal or the vertical component only was simulated, respectively. For the multi-directional analyses no component of movement was restricted along the base of the mesh. The first 20 s of the acceleration time-histories were simulated, with the accelerations applied incrementally along the bottom boundary. A modified Newton-Raphson scheme employing a sub-stepping stress point algorithm was the non-linear solver (Potts & Zdravković, 1999), while the generalised α-method of Chung & Hulbert (1993) with a spectral radius at infinity, ρ_∞, of 0.818 was used as the time-integration scheme. To achieve an accurate solution a time step of Δt=0.01 s was found to be small enough for the Lotung motion, whereas in the case of the Christchurch motion, due to the wider frequency content, Δt had to be reduced to a value of 0.003 s.

A two-surface bounding surface plasticity constitutive model which can numerically reproduce most of the significant features of sand behaviour under cyclic loading and realistically simulate liquefaction was used. The model is based on the Papadimitriou & Bouckovalas (2002) modified version of the original two-surface model proposed by Manzari & Dafalias (1997), extended so as to tackle cyclic loading and complex dynamic phenomena involving a range of cyclic strain amplitudes. The model has been implemented in ICFEP in generalized three-dimensional stress space (Taborda, 2011; Taborda et al., 2014) and includes several alterations which improve various aspects of its capabilities (a power law for the expression of the Critical State Line, an altered expression of the hardening modulus and the introduction of a secondary yield surface to improve the numerical stability).

Table 1. Model parameters for Fraser River Sand (Williams, 2014).

Model	Value	Model	Value	Model	Value
p'_{ref}	100 kPa	A_o	1	h_o	0.1
$(e_{CS})_{ref}$	0.84	m	0.065	γ	0.65
λ	0.03	p'_{YS}	1 kPa	e_{max}	0.946
ξ	0.66	B	422	α	1
M_c^c	1.376	a_1	0.44	β	-0.3
M_e^c	1	κ	2	μ	0
k_c^b	2.67	γ_1	0.0016	H_o	12600
k_c^d	1.67	ν	0.2	ζ	2

The model requires a total of 24 input parameters. Table 1 presents the parameters for Fraser River Sand established by Williams (2014). A total of 63 drained and undrained monotonic triaxial compression and extension element tests, as well as 21 cyclic drained and undrained direct simple shear tests were available for the calibration of the model (Ghafghazi, 2011; Thomas, 1992; Sriskandakumar, 2004). The meaning of each model parameter is explained in detail in Taborda (2011) and Taborda et al., (2014).

Results of Analyses

Mean Effective Stress Profiles

Figure 3 shows the mean effective stress profile with depth at the end of the dynamic motion for each of the six analyses conducted. Plotted in the same figure is the initial mean effective stress profile prior to the application of any loading. Clearly, the scaled horizontal component of the Lotung motion (denoted as LH) is strong enough to induce significant non-linearity along the whole depth of the deposit and eventually liquefaction down to about 26m depth. The scaled vertical component on its own (denoted as LV), however, although substantially strong, does not affect the effective stresses. As a direct consequence, when the two orthogonal components are combined in the analysis (LHV), the results are similar to those of the horizontal component only.

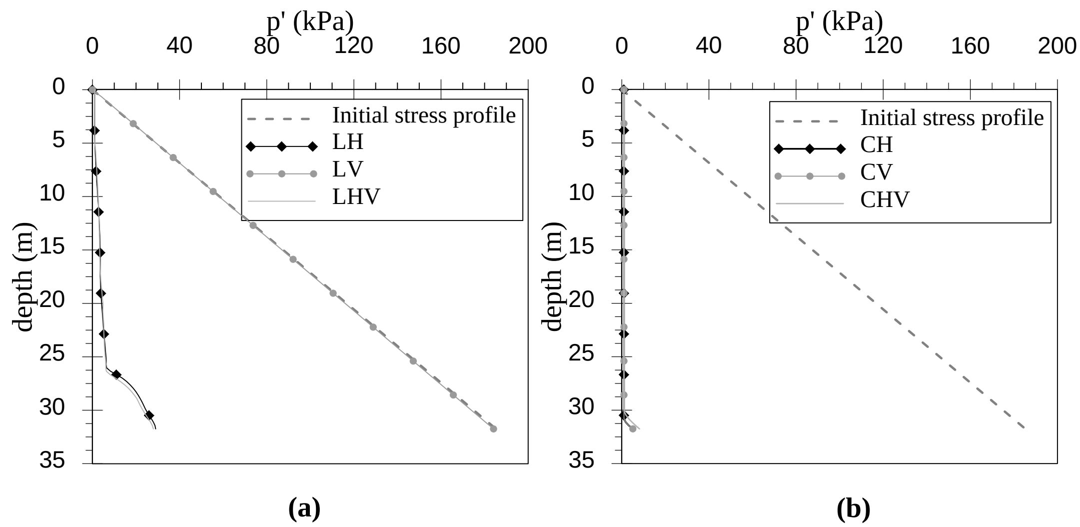

Figure 3. Initial and final mean effective stress profiles registered for the (a) Lotung and the (b) Christchurch seismic events

Similar to the above, the horizontal component of the Christchurch motion (denoted as CH) is sufficiently strong to liquefy the whole depth of the sand deposit. Contrary to the Lotung case, however, the analysis with the vertical component only (CV) predicts significant plastic response and liquefaction of the whole deposit. As anticipated, the multi-directional analysis (CHV) results in practically identical behaviour. It should be noted that the predicted depth of liquefaction for CH as well as LH agrees well with empirical calculations of the factor of safety against liquefaction (Idriss & Boulanger, 2008).

Mean Effective Stress and Excess Pore Pressure Ratio Time-Histories

The observations made above can be clearly seen in the mean effective stress and excess pore pressure time-histories at mid-depth of the deposit (Figure 4). Clearly, for the case of the Lotung seismic event the vertical excitation on its own (LV) results in no permanent changes in the effective stress and pore water pressures apart from high frequency oscillations as a result of total normal stress change. The horizontal excitation results in a progressive reduction of the effective stress with pore water pressures becoming equal to the initial vertical effective stress at 16 m depth at about 10.5 s of dynamic loading, simultaneously with the occurrence of the largest cycle in the acceleration time-history. No difference can be observed between the latter analysis and the multi-directional one in terms of variation of mean effective stress.

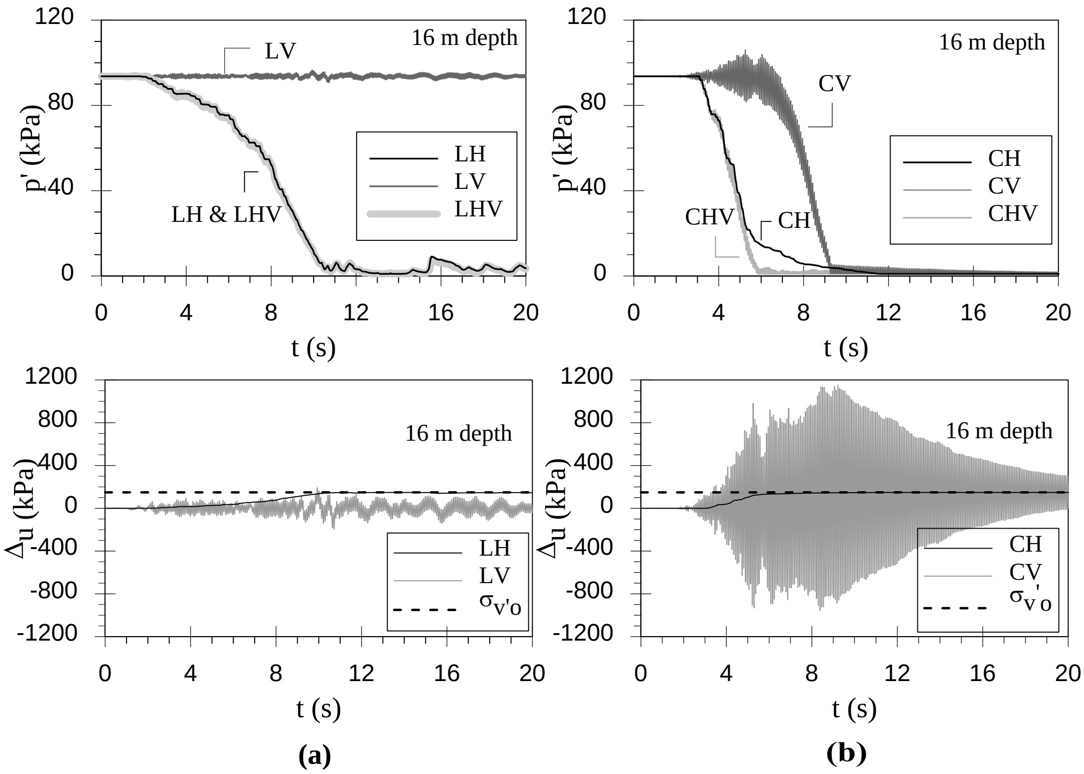

Figure 4. Mean effective stress and excess pore water pressure time-histories at mid-depth of the deposit for the (a) Lotung and the (b) Christchurch seismic events

Conversely, in the case of the vertical excitation of the Christchurch seismic event, the presence of substantial components in the input motion at a frequency range between 12 and 13 Hz, where the fundamental frequency of the deposit lies, and the subsequent amplification of these components by the deposit, results in plastic response and complete loss of strength at about 9 s, towards the end of the strong motion. This occurs slightly later compared to liquefaction due to S-waves, which can be attributed to the two mechanisms being very different as the stiffness of the deposit in compression is much higher, thus resulting in smaller deformations and therefore a more gradual increase of excess pore water pressures, Δu, compared to the case of the S-waves. Although high frequency oscillations are also present in the Δu time–histories, permanent excess pore pressure exists, with the final value of Δu oscillating around the initial vertical effective stress. As expected, even though the combined action of the two seismic components results in greater plasticity, the occurrence of complete loss of effective stress marginally precedes that of the analysis with the horizontal component only. This can be understood if one considers that the strong part of the motion for both components takes place at relatively similar instants, with the peak cycle of the vertical excitation taking place marginally earlier than that of the horizontal motion.

Stress Paths

The physical mechanism behind the above findings is easily understood by analysing the stress paths at mid-depth of the deposit, which are shown in Figure 5.

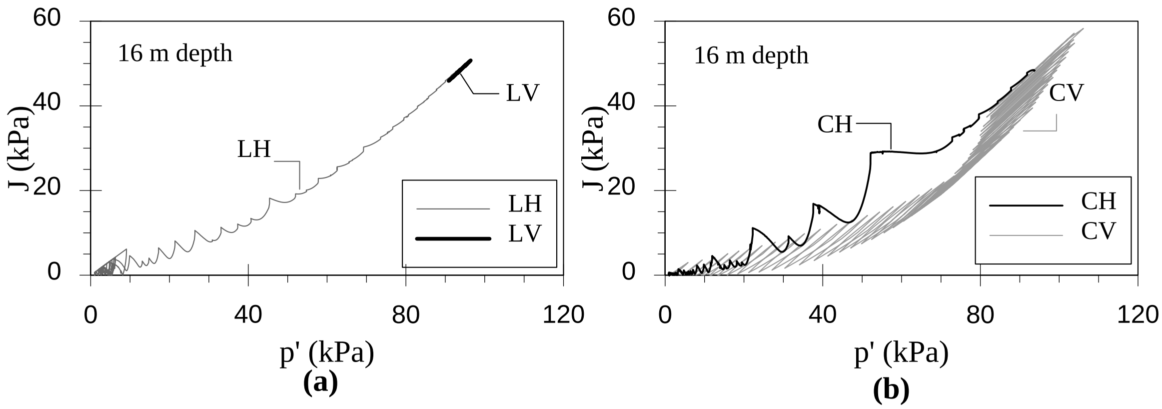

Figure 5. Stress paths in mean effective – deviatoric stress space (p' – J) for the (a) Lotung and the (b) Christchurch seismic events

When the Lotung vertical excitation is simulated on its own, the stress path is linear elastic with no variation in mean effective stress. However, in the case of the Christchurch vertical component, the loading is strong enough to induce substantial changes in normal effective stresses. As the model is formulated in the generalised stress space, compressible pore fluid is assumed and drainage can take place, the above leads to the development of significant deviatoric stresses which result in plastic strains and, therefore, in an increase in pore water pressures. The stress paths of the multi-directional analyses have not been included, as they are very similar to those of the analyses with the horizontal component only.

Conclusions

This numerical study focuses on the implications of vertical ground motion as well as multi-directional seismic loading on the liquefaction response of fully saturated sand deposits, placing emphasis on the frequency content of the input excitation. The results show that, contrary to the findings of previous studies, when the frequency content is rich in the range where the fundamental frequency of the deposit for P-waves lies, amplification of these components by the deposit, can lead to the development of significant deviatoric stresses which in turn can induce plasticity and, if strong enough, may lead to soil liquefaction. These findings indicate that the commonly adopted assumption of linear behaviour when compressional waves are simulated may not always be correct. Furthermore, when the two components (i.e. vertical and horizontal) are combined in the analysis, increased plasticity can be engaged. However, in the present study liquefaction in the multi-directional analysis occurs only marginally earlier compared to when only the horizontal ground motion is modelled.

Acknowledgments

The first author would like to gratefully acknowledge the financial support by EPSRC.

References

Bathe K-J. *Finite element procedures*. New Jersey: Prentice Hall, 1996.

*Chung J, Hulbert GM. A time integration algorithm for structural dynamics with improved numerical dissipation: the generalized-α method. J Appl Mech. 1993;**60**(2):371.*

*Elgamal A-W, Zeghal M, Tang HT, Stepp JC. Lotung downhole array. I: Evaluation of site dynamic properties. J Geotech Eng. 1995;**121**(4):350–362.*

Ghafghazi M. *Towards comprehensive interpretation of the state parameter from cone penetration testing in cohesionless soils*. PhD thesis, Department of Civil Enginerring, University of British Columbia, 2011.

*Hardin BO, Richart Jr FE. Elastic wave velocities in granular soils. J Soil Mech Found Div. 1963; **89**(SM1):33–65.*

Idriss IM, Boulanger RW. *Soil liquefaction during earthquakes*. Oakland,California, USA: Earthquake Engineering Research Center, College of Engingeering, University of California; 2008.

International Code Council. *Uniform building code*, 1996.

Ishihara K. *Soil behaviour in earthquake geotechnics*. Oxford Engineering Series. Oxford, Oxford University Press., 1996.

*Manzari MT, Dafalias YF. A critical state two-surface plasticity model for sands. Géotechnique. 1997;**47**(2):255–272.*

*Papadimitriou AG, Bouckovalas GD. Plasticity model for sand under small and large cyclic strains: A multiaxial formulation. Soil Dyn Earthq Eng. 2002;**22**(3):191–204.*

Potts DM, Zdravković L. *Finite element analysis in geotechnical engineering: theory*. London: Thomas Telford, 1999.

*Rathje EM, Faraj F, Russell S, Bray JD. Empirical Relationships for Frequency Content Parameters of Earthquake Ground Motions. Earthq Spectra. 2004;**20**(1):119–144.*

Shiomi T, Yoshizawa M. Effect of multi-directional loading and initial stress on liquefaction behaviour. In: Eleventh World Conference on Earthquake Engineering. Acapulco, Mexico: Elsevier Science Ltd, 1996.

Sriskandakumar S. *Cyclic loading response of Fraser River Sand for validation of numerical models simulating centrifuge tests.* MSc thesis, Department of Civil Enginerring, University of British Columbia, 2004.

Taborda DMG. *Development of constitutive models for application in soil dynamics*. PhD thesis, Department of Civil & Environmental Engineering, Imperial College London, 2011.

*Taborda DMG, Zdravković L, Kontoe S, Potts DM. Computational study on the modification of a bounding surface plasticity model for sands. Comput Geotech. 2014;**59**:145–160.*

Thomas J. *Static, cyclic and post liquefaction undrained behaviour of Fraser River Sand.* MSc thesis, Department of Civil Enginerring, University of British Columbia, 1992.

Williams JD. *Modelling of anisotropic sand behaviour under generalised loading conditions*. MSc thesis, Department of Civil & Environmental Engineering, Imperial College London, 2014.

*Yang J, Sato T, Savidis S, Li XS. Horizontal and vertical components of earthquake ground motions at liquefiable sites. Soil Dyn Earthq Eng. 2002;**22**(3):229–240.*

Zienkiewicz OC, Bicanic N, Shen FQ. *Earthquake input definition and the transmitting boundary conditions*. In: St Doltnis I, ed. Advances in computational nonlinear mechanics., 1988:109–138.