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Unified Correlation between SPT-N and Shear Wave Velocity for all Soil Types

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ABSTRACT

The shear wave velocity (Vs) of sediments plays a key role in seismic wave amplification and is required in site response analysis. Such information is usually lacking during field exploration, but standard penetration test blow count (N) is mostly obtained. Therefore, several studies have established empirical correlations between N and Vs for engineering use. These empirical correlations, however, vary significantly in terms of model form and parameters. A unified empirical correlation is developed in this study through the use of the Engineering Geological Database for the Taiwan Strong Motion Instrumentation Program, which contains grain-size distribution, void ratio, water content, specific gravity, unit weight, liquid limit, and plasticity index (PI) in addition to the Vs and N of strata. The influence of overburden pressure, fines content (FC), PI, and soil types on small-strain behavior (i.e., Vs) and large-strain behavior (i.e., N) is discussed. A unified correlation between Vs and N that is dependent on overburden pressure, FC, and PI is proposed through the conditional prediction approach.

Introduction

A key property required to effectively estimate the seismic response of a site is small-strain shear modulus Go, which is often computed by measuring shear wave velocity Vs and mass density ρ as follows:

$$Go= \rho Vs^2. \qquad (1)$$

Geophysical investigations are typically performed to measure the Vs profile. However, these measurements are not always common because of the additional cost of the field investigation. Correlations between Vs and standard penetration test (SPT) blow count (N), conditioned on the geologic setting and site stratigraphy, are potentially useful for the above situation.

Numerous relations between N and Vs for Taiwan regions have been established in the previous study (Kuo *et al.*, 2011). These empirical correlations, however, vary significantly in terms of model form and are limited to a specific soil type (e.g., sand or clay). The development of these correlations also lacks theoretical support. Therefore, with a theoretical basis, this study aims to establish a unified correlation between N and Vs that can be applied to sand, silt, and clay. The Engineering Geological Database for the Taiwan Strong Motion Instrumentation Program

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(EGDT) (Kuo *et al.*, 2011) is utilized in the analysis. The factors that can change the small-strain properties Go are reviewed. Based on a previous study on laboratory test data, a model form that describes small-strain properties (i.e., Vs) and large-strain measurements (i.e., N) is proposed. The influence of effective overburden pressure (σ_o’), fines content (FC), plasticity index (PI), and over consolidation ratio (OCR) on small-strain properties and large-strain measurements is discussed based on the regression analysis results. Lastly, a unified correlation between Vs and N that is dependent of these parameters is proposed through the conditional prediction approach.

Theoretical Form of Vs and N

Fundamental Functional Form

The most common functional form for the relations of Go proposed in literature is

$$\mathrm{Go} = \mathrm{AF(e)}(\sigma_0{}')^{n}, \tag{2}$$

where $\sigma_0{}'$is the effective confining (or vertical) stress, F(e) is the function of void ratio, and constants A and n are determined by statistical regression of a data set.

A summary by Ishihara (1996) indicates that n is mostly 0.5 for both sand and various types of clays (i.e., n = 0.25 for Vs according to Eq. (1)). However, as strain amplitude increases, n increases according to laboratory test data from 1/3 to 1.0 as shear strain $\gamma < 10^{-4}\%$ increases to $\gamma = 10^{-2}\%$ (i.e., the exponent term of $\sigma_0{}'$ changes from 1/6 to 0.5 as γ increases from $10^{-4}\%$ to $10^{-2}\%$ for Vs). The change in n indicates that confining stress influences small- and large-strain soil properties differently. Therefore, the exponent n is strain-dependent, which implies that small-strain property (e.g., Vs or Go) and large-strain measurement (e.g., N) need to be corrected by confined pressure differently. As a result, confined pressures should be included when establishing the correlation between N and Vs, as suggested by Brandenberg *et al.* (2010).

Other Factors that Influence Functional Form

Hardin and Black (1968) discussed the model form of Go for normally consolidated (NC) and overly consolidated (OC) clays and suggested that

$$\mathrm{Go} = \mathrm{AF(e)OCR}^{PI/160}(\sigma_0{}')^{n}. \tag{3}$$

The equation implies the following. First, for a non-plastic or low-PI soil (e.g., sand or silt), Go is independent of OCR. Second, given OCR = 1 (NC clay), Eq. (3) yields Eq. (2) and exponent n is a fixed number regardless of PI. Viggiani and Atkinson (1996) reported that impact of confined stress on Go is uncoupled with OCR as

$$\mathrm{Go} = \mathrm{B}(\sigma_0{}')^{n}\mathrm{OCR}^{m}. \tag{4}$$

However, B, n, and m are dependent on PI as revealed by laboratory test results. Kawaguchi and Tanaka (2008) recently proposed a theoretical form of Go as follows:

$$\mathrm{Go} = 20000 \cdot \mathrm{w}_L{}^{-0.8} \cdot (\tfrac{2}{3} OCR)^{0.2} \cdot (\tfrac{1+OCR^{0.5}}{3})^{0.6} \cdot \ (\sigma_0')^{0.8}, \tag{5}$$

where $\mathrm{w_L}$ is the liquid limit, which is correlated to PI. Equation (5) indicates that the effect of OCR and PI (or $\mathrm{w_L}$) on Go is uncoupled unlike that in Eq. (3) where the influence of OCR and PI on Go is coupled.

Proposed Model Form

Based on the review of previous studies, two possible regression models of Go are proposed as follows:

$$\text{Model 1: } \mathrm{Go} = \mathrm{A} \cdot \mathrm{PI}^{m} \cdot \mathrm{OCR}^{l} (\sigma_0')^{n}, \tag{6}$$
$$\text{Model 2: } \mathrm{Go} = \mathrm{A} \cdot \mathrm{OCR}^{(\mathrm{a} \cdot \mathrm{PI})} (\sigma_0')^{n}. \tag{7}$$

The effect of PI and OCR on Go is uncoupled in Model 1, whereas it is coupled in Model 2. Considering that the coupling effect between PI and OCR is still under debate, this study adopts Model 1 for the development of the correlation between N and Vs because of its simplicity. Meanwhile, FC can also change the blowout, and its effect is usually accounted for when performing liquefaction analysis. Therefore, the proposed general model for Vs and N is further modified as

$$\mathrm{Vs} = c_0 \cdot (\sigma_0')^{c_1} \cdot FC^{c_2} \cdot \mathrm{PI}^{c_3} \cdot \mathrm{OCR}^{c_4}, \tag{8}$$
$$\mathrm{N} = b_0 \cdot (\sigma_0')^{b_1} \cdot FC^{b_2} \cdot \mathrm{PI}^{b_3} \cdot \mathrm{OCR}^{b_4}. \tag{9}$$

As discussed earlier, confining stress influences small- and large-strain soil properties differently (i.e., c_1 and b_1 are different). Similarly, the influence of FC, PI, and OCR on small- and large-strain behavior may also be different. This issue will be discussed later based on the regression analysis.

Conditional Prediction Approach

The regression analysis of Vs with conditional measurements of N based on Kishida and Tsai (2015) is described in this section. First, regression analyses are performed separately to determine the coefficients in Eqs. (8) and (9). Second, the relationship between Vs and N (Vs conditional on N) is established based on the correlation of the residual in Eqs. (8) and (9). As a simple example to illustrate the conditional prediction approach, we use the following two regression models that only include the confined stress term.

$$\ln N = b_0 + b_1 \ln \sigma_o' + \varepsilon_N \ , \tag{10}$$
$$\ln V_s = c_0 + c_1 \ln \sigma_o' + \varepsilon_{Vs} \ , \tag{11}$$

where ε_N and ε_{Vs} are the residuals after regression analysis and follow the normal distributions with mean = 0 and standard deviation of σ_N^2 and σ_{Vs}^2, respectively. The correlation between ε_N and ε_{Vs} is ρ_{NVs}. Conditional prediction of lnVs given lnN is expressed as follows:

$$E\left[\ln Vs \mid \ln N\right] = E\left[\ln Vs\right] + \sigma_{Vs}\rho_{NVs}\frac{\varepsilon_N}{\sigma_N} = c_0 - b_0\frac{\sigma_{Vs}}{\sigma_N}\rho_{NVs} + \frac{\sigma_{Vs}}{\sigma_N}\rho_{NVs}\ln N + \left(c_1 - b_1\frac{\sigma_{Vs}}{\sigma_N}\rho_{NVs}\right)\ln\sigma'_o \tag{12}$$

$$\sigma^2_{Vs|N} = \sigma_{Vs}^{2}\left(1 - \rho^2_{NVs}\right). \tag{13}$$

Therefore, the following equation is obtained.

$$E\left[\ln Vs \mid \ln N\right] = \beta_0 + \beta_1 \ln N + \beta_2 \ln\sigma'_o, \tag{14a}$$

where

$$\beta_0 = c_0 - b_0\frac{\sigma_{Vs}}{\sigma_N}\rho_{NVs}, \tag{14a}$$

$$\beta_1 = \frac{\sigma_{Vs}}{\sigma_N}\rho_{NVs}, \tag{14a}$$

$$\beta_2 = c_1 - b_1\frac{\sigma_{Vs}}{\sigma_N}\rho_{NVs}. \tag{14a}$$

The proposed conditional approach can be extended in a similar manner when more terms (e.g., FC, PI, and OCR in Eqs. (8) and (9)) are added to Eqs. (10) and (11).

Database for Regression Analysis

The basic data provided in EGDT include stratum description, results of soil physical property tests (such as grain-size distribution, uniformity coefficient, coefficient of gradation, void ratio, water content, specific gravity, unit weight, liquid limit, and PI), soil classification, P- and S-wave velocities, and SPT-N values. EGDT provides sufficient information for the regression analysis in addition to the measurement of N and Vs. However, OCR is unavailable in EGDT. Therefore, OCR is approximately estimated by Eq. (5) in this study. The influence of this assumption will be explored later in the paper. Effective vertical stresses are calculated with the given depth, unit weight, and water table elevation. Groundwater elevation is sometimes not recorded for some borings. In such a case, the p-wave velocity profile is utilized to identify the approximate elevation of the groundwater table. An abrupt transition from p-wave velocity lower than 500 m/s to higher than 1500 m/s is typically apparent in boring logs, clearly indicating the position of the groundwater table.

In EGDT, SPT-N is measured by an automatic hammer falling system every 1.5 m (every 3 m to 5 m for gravel layers) or at the depth of notable discontinuity during drilling. The N is corrected to N_{60} by assuming the measured energy ratio of 70%. Only SPT-N less than 50 is utilized for the regression analysis. P- and S-wave velocities are measured with a suspension PS logger system. Velocity measurement is generally performed every 0.5 m, except for several drillings in the first and second years, in which velocity is measured for every 1 m. A total of 3684 data sets from 334 sites that include Vs, N (<50), σ_o', FC, PI, and estimated OCR are utilized for regression. The distribution of data is presented in Figure 1. For non-plastic soil, PI is set as a unit; for soil without FC, FC is also set as a unit. As shown in Figure 2, Vs is approximately linear against the model parameters in log–log space, which indicates that modeling by Eqs. (8) and (9) is sufficient.

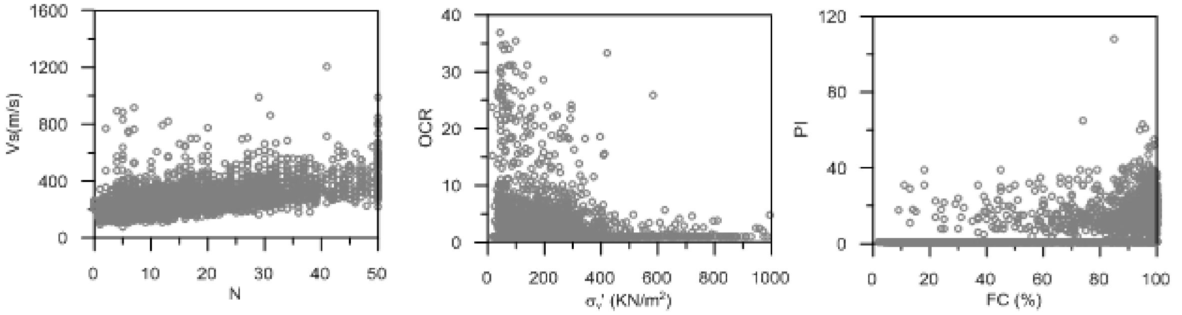

Figure 1. Distribution of data sets

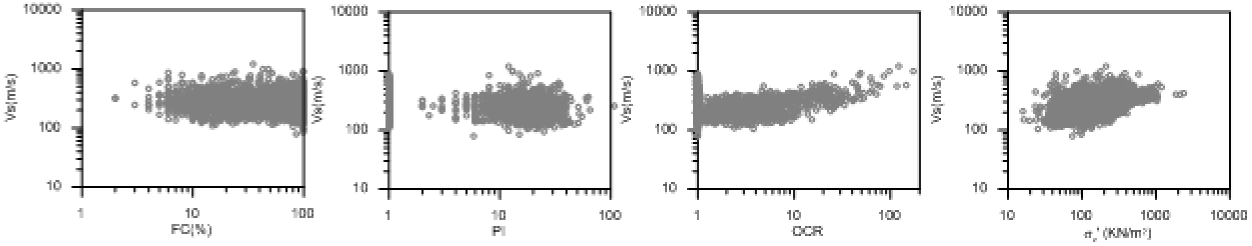

Figure 2. Vs against model parameters in log–log space

Regression Analysis Results

Small- and Large-Strain Behavior

Table 1 provides a summary of the regression result obtained with Eqs. (8) and (9). The exponent term of FC and PI is negative, whereas that of σ_o’ and OCR is positive. The positive value indicates that N or Vs increases as OCR and σ_o’ increase, and the negative value indicates that N or Vs decreases as FC and PI increase. However, the value of the exponent term is different for N and Vs. The exponent term of σ_o’ for Vs (small-strain property) is approximately 0.25, whereas that for N (large-strain measurement) is approximately 0.5. This result is consistent with that of several previous laboratory studies summarized in Ishihara (1996), that is, as strain amplitude increases, the exponent of σ_o’ changes from 1/6 to 0.5.

Different exponent values of predictor variables mean different degrees of influence by these variables on N and Vs. The degrees of various influence can be quantified by the exponent ratio between N and Vs, in which a high value implies the most different effect on large- and small-strain behavior. Therefore, as shown in Table 1, FC influences N and Vs most differently followed by σ_o’, PI, and OCR. Such difference should be considered when developing the correlation between N and Vs. With σ_o’ as an example, the obtained exponent ratio is approximately 2. Therefore, if Vs is conditioned on N (or N is normalized by Vs similar to the modulus reduction curve where shear modulus G is normalized by Gmax at small strain), the influence of σ_o’ is not negligible. In other words, the prediction model of Vs based on N should include the σ_o’ term, as reported by Brandenberg *et al.* (2010). Similarly, FC and PI should be included in the model because they also influence small- and large-strain behavior differently. Only the ratio of OCR is approximately 1. Thus, the prediction model of Vs based on N can possibly ignore the OCR term.

Table 1. Results of the regression analysis of model 1

	Intercept	Exponent				R^2	□	ρ_{NVS}
		σ_o’	FC	PI	OCR			
N	0.90	0.58	−0.27	−0.37	0.40	0.50	0.61	0.32
Vs	4.59	0.26	−0.08	−0.18	0.32	0.42	0.28	
Ratio	-	2.25	3.43	2.05	1.26	-	-	-

Influence of FC

The developed correlation between N and FC and Vs and FC can also be utilized to evaluate the impact of FC on Vs and N. The exponent of FC for N is higher than that for Vs, indicating that FC has more influence on N. In liquefaction potential analysis, N and Vs require correction to consider the impact of FC. Corrected (or equivalent) N or Vs is typically higher than measured N or Vs for soil with certain FC because high FC results in low measured N and Vs or can increase the resistance of liquefaction. The negative exponent of FC obtained in this study is consistent with the concept of correction for FC in the liquefaction analysis. We also quantitatively compare FC correction from our regression analysis with that proposed by previous studies.
Figure 3(a) shows a comparison of FC-corrected N (Ncs) by Idriss and Boulanger (2008), Seed *et al.*, (2003), and Youd *et al.* (2001) for various FCs given measured N = 5, 10, 15. Ncs in this study is calculated as follows:

$$Ncs=N/FC^{-0.274}. \tag{15}$$

Given measured N = 5 and 10, the Ncs in this study is in the range of the Ncs obtained by the other studies. For N = 15, the Ncs in this study is slightly larger than the Ncs in the other studies. The previous studies mostly corrected N with FC up to 35% to 40%. By contrast, the result of this study indicates that N may be further corrected for a higher FC by considering mechanical correction during measuring.

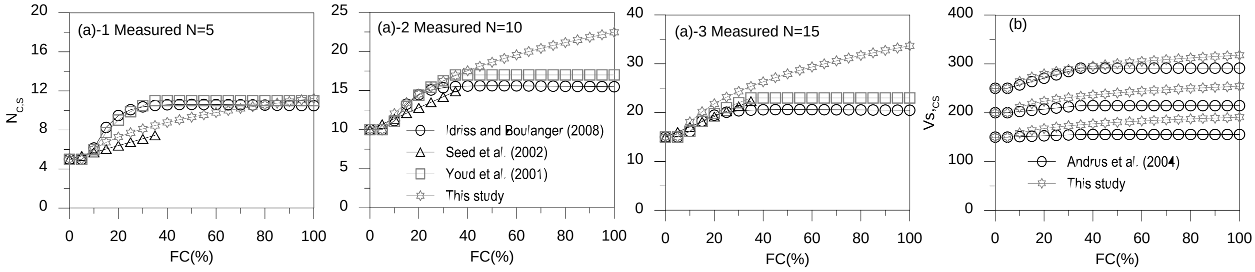

Figure 3. Comparison of (a) corrected N for various FCs given measured N = 5, 10, 15 and (b) corrected Vs for various FCs given measured Vs = 150, 200, 250 m/s

We also compare FC-corrected Vs ($Vs_{,CS}$) in this study with that corrected by Andrus *et al.* (2004) for various FCs given three measured Vs. As shown in Figure 3(b), the $Vs_{,CS}$ in this study is slightly higher than that in Andrus *et al.* (2004) for low measured Vs but agrees well with that in Andrus *et al.* (2004) for measured Vs = 250 m/s. Similar to the impact of FC on N, Vs may be further corrected for a high FC, as indicated by our regression model.

Unified Model

A unified empirical model is proposed according to the previously described conditional prediction procedure (Eqs. (10) to (14a)) by using the regression result listed in Table 1, that is,

$$\mathrm{lnVs} = 4.46 + 0.15\mathrm{lnN} + 0.17\mathrm{ln}\sigma_0{}' - 0.04\mathrm{ln}FC - 0.12\mathrm{lnPI} + 0.26\mathrm{lnOCR}. \tag{16}$$

The standard deviation of $\sigma_{\mathrm{lnvs/N}}$ is 0.26, which is approximated as $\sigma_{\mathrm{vs/N}}$ = 63 m/s given mean Vs = 270 m/s. In Kuo *et al.* (2011), the data set similar to the one used in this study was grouped into clay and sand for regression analysis, and the obtained σ of individual Vs-N model was 67 and 77 m/s, respectively. σ in the present study is lower than that by Kuo although we used the entire dataset in the regression analysis. The model that includes additional prediction parameters such as FC and PI significantly improve prediction accuracy. The model can also be applied to more general conditions for estimating Vs by N and is not limited to a specific soil type.

Considering that OCR is not directly provided in the database and is estimated indirectly, we also evaluate model performance if OCR is excluded from the model. The result shows that removing OCR from the model has a limited effect because σ is slightly increased from 63 m/s to 68 m/s given mean Vs = 270m/s. The model based on σ_o', FC, and PI can sufficiently predict Vs although OCR is typically unavailable in practice. This result may be due to the fact that the impact of OCR on small- and large-strain behavior is similar, as discussed earlier. Therefore, the following equation can be adopted if OCR is unavailable.

$$\mathrm{lnVs} = 4.52 + 0.22\mathrm{lnN} + 0.11\mathrm{ln}\sigma_0{}' - 0.03\mathrm{ln}FC + 0.02\mathrm{lnPI} \tag{17}$$

If the model form only includes σ_o' as typically modeled by others, σ becomes high (Vs = 94 m/s). Nevertheless, it is still lower than the σ obtained by Kuo *et al.* (2011) (approximately 108 m/s). This result may be attributed to the fact that we adopted a conditional prediction approach and used σ_o' instead of depth in the model.

Conclusions

Numerous relations between N and Vs have been proposed for practical purposes in earthquake engineering. These empirical correlations, however, vary significantly in terms of model form and parameters. Without a theoretical or experimental basis, they are developed only for a specific site and soil type. Vs and N represent small- and large-strain behavior, respectively. Such difference should be considered when a model is developed. Therefore, a unified empirical correlation model was developed in this study using a conditional prediction approach to correlate small- and large-strain behavior. The developed model can be applied to clay, silt, and sand. EGDT with 3684 data sets was utilized to develop the model.

The main factors that can change the small-strain property include σ_o', FC, PI, and OCR according to a previous study on laboratory test data. Therefore, we developed a simple model form that includes these parameters to estimate small-strain property and applied it to large-strain measurement similarly. The influence of these parameters on small-strain properties (i.e., Vs) and large-strain measurements (i.e., N) was discussed according to the regression result. Different exponent values of predictor variables mean different degrees of influence by these parameters on large- and small-strain behavior. As indicated by the exponent ratio between N and Vs, FC influences N and Vs most differently, followed by σ_o, PI, and OCR. The exponent ratio of σ_o' between N and Vs is approximately 2, which is consistent with that in previous studies. The change in N and Vs by FC in the regression result agrees well with the fine correction recommended in the liquefaction potential analysis.

Lastly, a unified correlation model between Vs and N that is dependent on σ_o', FC, PI, and OCR was established through a conditional prediction approach. The model includes additional prediction parameters, such as FC, OCR, and PI, which significantly improve prediction accuracy according to the reduction in standard deviation. The proposed model can be applied to more general conditions and is not limited to a specific soil type. However, the correlation has only been tested for soils in Taiwan and has yet to be verified for other soils in other regions.

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