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# Identification and Extraction of Surface Waves from Three-Component Seismograms Based on the Normalized Inner Product

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## ABSTRACT

In this work, we propose a new time-frequency analysis procedure to identify and extract Rayleigh and Love waves from three-component seismograms. Exploiting the advantage of the absolute phase preservation by the Stockwell Transform, we construct time-frequency filters to extract waves based on the 'Normalized Inner Product' (NIP). Rayleigh and Love waves can be identified through the NIP between the Stockwell Transforms of the horizontal and vertical displacement components. The novelty and advantage of the proposed procedure is that it does not require making any assumptions regarding the direction of propagation of the surface waves which is a byproduct. The procedure has been successfully tested with real signals, in particular to extract Rayleigh and Love waves for aftershocks of the 1999 Chi-Chi earthquake. With the proposed procedure we found different directions of propagation for retro-grade and pro-grade Rayleigh waves, which might suggest that they are generated by different mechanisms.

## Introduction

The identification of surface waves is important because they carry information about the surficial soil layers and they impact man-made structures. Methods for identifying surface waves are based on the two main characteristics of such waves: (1) plane and type of polarization, and (2) frequency-dependent phase velocities (dispersion). Identification of Rayleigh wave phases has been traditionally performed by means of Complex Trace Analysis (CTA) (Vidale, 1986, René et al. 1986, Li & Crampin 1991, Baker & Stevens 2004). It uses time-varying polarization characteristics to differentiate between waves. However, since dispersed waves (such as surface waves) may be effectively analyzed in terms of narrow-band wave packets, an efficient extraction technique is needed. Considering the CTA, we need to choose frequency ranges of interest and assume the direction of propagation a priori.

Consequently, a time-frequency polarization analysis seems to be a more appropriate alternative. Pinnegar (2006) and Galiana-Merino (2011) have constructed filters to exclude or extract Rayleigh waves based on the instantaneous reciprocal ellipticity defined in the time-frequency domain. The filtering criteria need to be independent of the direction of the two horizontal components of the signal. This is the case for the instantaneous reciprocal ellipticity. However, the reciprocal ellipticity criterion alone does not work well if the time history contains Rayleigh waves with both retro-grade and pro-grade motion. Such a case has been observed in recordings of the aftershock 1803 of the 1999 Mw 6.2 Chi-Chi earthquake (Wang et al. 2006).

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The proposed method to detect and extract surface waves from three-component recorded seismograms overcomes these difficulties. We exploit the advantage of the absolute phase preservation of the Stockwell Transform (Stockwell et al. 1996), and we construct time-frequency filters to extract waves based on the 'Normalized Inner Product' (NIP). Since the NIP is the time-frequency counterpart of the correlation, Rayleigh and Love waves can be identified based on the value of the NIP between the Stockwell Transforms of the horizontal and vertical displacement components. No assumption on the direction of propagation of the surface waves is required, but such direction is a byproduct of the proposed computational procedure.

## Time-Frequency Polarization Analysis

Three-component signals can be thought as a superposition of sinusoids oscillating in the $x$-, $y$-, and $z$- directions, corresponding to a superposition of ellipses. When the analyzed components are in-phase, we are considering linear polarization. When the components are out-of-phase and have different amplitudes, elliptical polarization is involved. In seismic signals, the polarization state of Rayleigh waves is elliptical, whereas body and Love waves are linearly polarized. The information contained in the Fourier Spectra of the $x$-, $y$- and $z$- components of the signal can be considered as spectral components of the ellipses. The same type of reasoning can be used with windowed Fourier (e.g. Stockwell) transforms to get time-varying spectral elliptical components. The Stockwell transform $S(t,f)$ of a time varying function $h(t)$ can be expressed in the following form (Stockwell et al. 1996):

$$S(t,f) = \int_{-\infty}^{\infty} h(\tau) \frac{|f|}{\sqrt{2\pi}} \exp\left[-\frac{(t-\tau)^2 f^2}{2}\right] \exp[-2\pi i f \tau]\, d\tau \tag{1}$$

The time-frequency elliptical elements of the polarization ellipse, which describe the contribution of the $f$-th frequency to the total signal are defined in (Pinnegar 2006) for the Stockwell transform. Among them, the instantaneous reciprocal ellipticity has been used to discern waves contained in a seismic signal (Pinnegar 2006, Galiana-Merino 2011). Filters are constructed and applied in the time-frequency domain, and then inverse Stockwell transform is used to recover the filtered time-domain signal. The inverse Stockwell transform is exactly obtained in two steps: the Fourier Transform is obtained by integrating the Stockwell transform over time, and then Fourier Transform is inverted (Stockwell et al. 1996).

## Computation of the Direction of Propagation

Once the polarization filtering is done, we can find the angle of propagation $\theta$ of the Rayleigh waves, using the filtered signals. One way to do this is rotating the horizontal filtered signals with trial back-azimuths until finding the best correlation coefficient between the "Radial" component and the Hilbert Transform of the Vertical component. We can avoid this iterative process, and make a direct computation of the angle $\theta$, using the following relation:

$$\tan(\theta) = \frac{\sum_i^N E_F(i)\hat{V}_F(i)}{\sum_i^N N_F(i)\hat{V}_F(i)} \qquad \theta \neq \frac{\pi}{2}, \frac{3\pi}{2} \tag{2}$$

where $E_F$ and $N_F$ are the filtered East and North components respectively, and $\hat{V}_F$ is the filtered Vertical component shifted 90 degrees. Equation (2) gives directly the back-azimuth.

## Extraction of Rayleigh Waves

In this section we apply the procedure for identification and extraction of surface waves produced by an aftershock of the ChiChi earthquake in the West Coastal Plain in Taiwan, and that occurred on 20 September 1999 at 1803 UTC with a magnitude of Mw 6.2. This aftershock is very useful to test the surface wave extraction method, since it produced very strong and clear surface waves in the plain. Figure 1shows a map with the directions of the Rayleigh wave propagation estimated by Wang et al. (2006), where the location of the epicenter is also indicated. The black circle in Figure 1 specifies the location of station TCU116, which will be considered for this example.

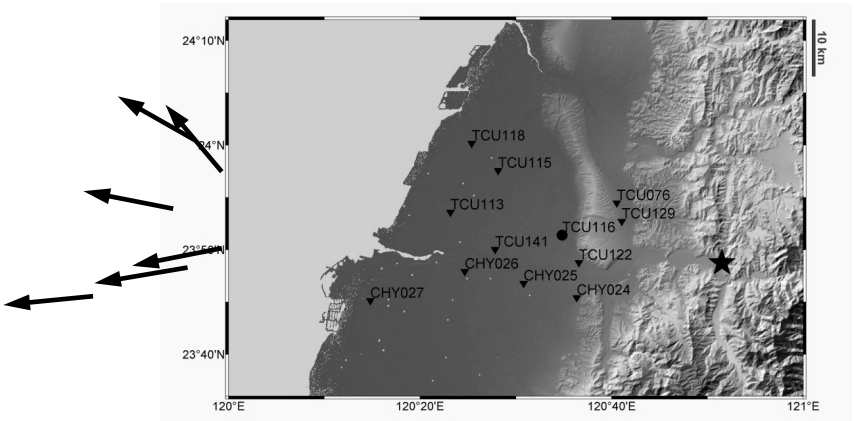


Figure 1. Map illustrating the location of the stations on the West Coastal Plain considered in this study. For some stations the arrows indicate the direction Rayleigh wave propagation estimated by Wang et al. (2006). The star indicates the location of the epicenter of the event. The black circle indicates the location of station TCU116.

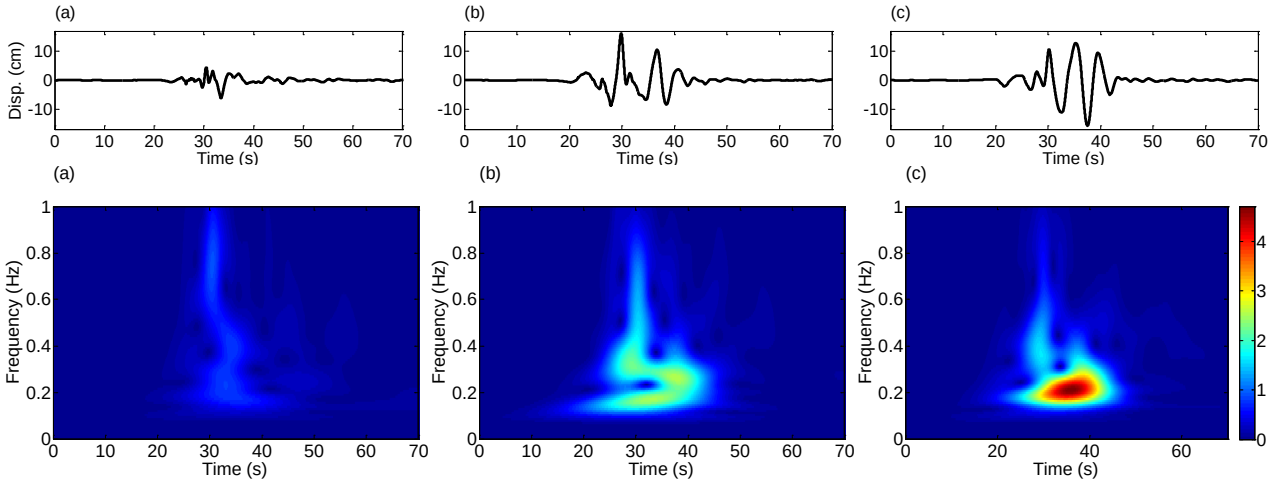


Figure 2. *NEV* components of the displacement history recorded during the aftershock at station TCU116 (top) and Stockwell transforms (bottom): (a) North component, (b) East component, (c) Vertical component.

The *NEV* components of the displacement time history recorded at this station during the aftershock are shown in Figure 2 (top). The displacements were obtained by independently bandpass filtering and integrating twice the components of the acceleration histories. Next, the Stockwell Transforms of the displacements histories are computed, and their amplitudes are shown in Figure 2 (bottom). Based on the instantaneous reciprocal ellipticity $\varphi$, computed as in (Pinnegar 2006) and ranging from 0 to 1, we construct a filter to exclude waves for which $\varphi \leq 0.5$. The results of the application of the filter to the Stockwell transforms are shown in Figure 3 (top), illustrating clearly the areas that the filter keeps and excludes. The filtered transforms are inverted, and the filtered time domain signals are shown in Figure 3 (bottom).

Now we use these extracted waves to compute the angle of propagation of the Rayleigh waves using Eq. (2). The computed angle is $\theta$=255.93 deg. which is in agreement with the predictions of Wang et al. (2006) for the propagation of surface waves in the plain (Fig. 1). With this computed angle, the horizontal *NE* filtered components are rotated to obtain components in the "Radial" and "Transverse" directions. The vertical component is then shifted 90 degrees and both components are compared in Figure 4. We can observe that the Radial and shifted Vertical components are in phase starting at about 30 seconds, but before this instant the extracted waves do not seem to resemble the characteristics of a Rayleigh wave. The correlation coefficient between the Radial and shifted Vertical components of the extracted waves is 0.7.

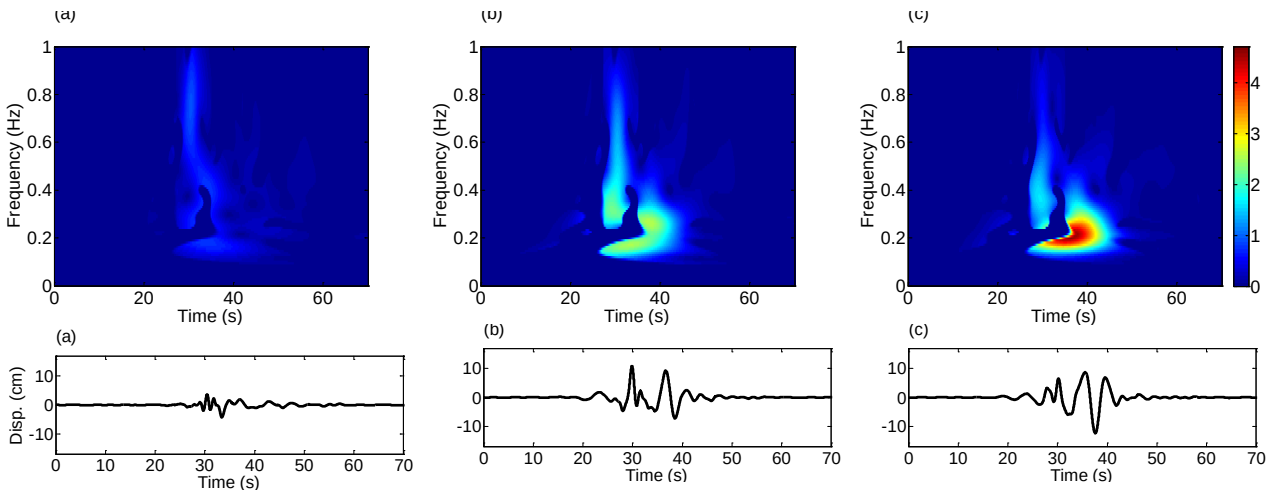


Figure 3. Filtered Stockwell transforms of the *NEV* displacements during the aftershock at station TCU116 (top) and Filtered NEV components of displacement history (bottom): (a) North component, (b) East component, (c) Vertical component.

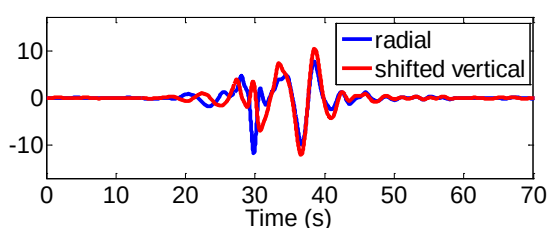


Figure 4. Comparison between the filtered Radial and shifted Vertical components of the displacement history recorded at station TCU116. The Vertical component is shifted 90 degrees.

## The Normalized Inner Product

As shown in the previous section, the instantaneous reciprocal ellipticity is not as effective as expected to extract surface waves. It is due to the fact that, in the case of the West Coastal Plain in Taiwan (Wang et al., 2006), the structure of the basin allows for the generation of both prograde and retrograde waves. In (Galiana-Merino, 2011) it is suggested that the instantaneous phase difference between the Radial and Vertical components can be used to discern between these two types of motion. In our case, the phase to be used to compute the instantaneous phase difference would be the phase of the Stockwell transform. However, the method proposed in (Galiana-Merino, 2011) requires knowledge of the angle giving the radial direction in order to extract the Rayleigh waves, and in most cases this angle is not available. We thus propose a new criterion (Normalized Inner Product) to extract Rayleigh waves from the signal without specifying their direction of propagation. It is defined as (Meza-Fajardo et al. 2015):

$$NIP = \frac{\mathrm{Re}\{S_R\}\mathrm{Re}\{\hat{S}_V\} + \mathrm{Im}\{S_R\}\mathrm{Im}\{\hat{S}_V\}}{|S_R||\hat{S}_V|} \tag{3}$$

where $S_R$ is the Stockwell transform of the Radial component of displacement, and $\hat{S}_V$ is the Stockwell transform of the Vertical component, shifted 90 degrees. $S_R$ is computed from the Stockwell transforms of the *NEV* components of displacement, using the instantaneous angle $\theta_I$:

$$\theta_I = \tan^{-1}\left(\frac{\mathrm{Re}\{S_E\}\mathrm{Re}\{\hat{S}_V\} + \mathrm{Im}\{S_E\}\mathrm{Im}\{\hat{S}_V\}}{\mathrm{Re}\{S_N\}\mathrm{Re}\{\hat{S}_V\} + \mathrm{Im}\{S_N\}\mathrm{Im}\{\hat{S}_V\}}\right) \tag{4}$$

In Figure 5(a), the NIP for the real signal recorded at station TCU116 is shown. We can identify areas where the NIP is close to 1 and other areas where the NIP is close to -1. To extract the retrograde Rayleigh waves, we need to exclude the (dark) areas where the NIP is close to -1. For comparison, the instantaneous reciprocal ellipticity is also shown in Figure 5(b). We can conclude from the comparison that the NIP is a better criterion to create filters and extract the Rayleigh waves, since it is more stable over the $(t, f)$ domain. Figure 5(b) shows that the instantaneous reciprocal ellipticity has more variations in the Stockwell transform domain.

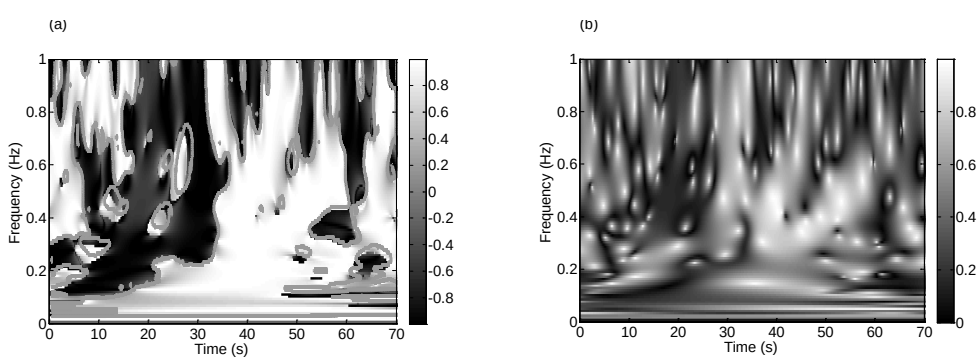


Figure 5. Comparison between instantaneous reciprocal ellipticity and NIP criteria.
(a) Normalized inner product of radial and phase advanced vertical component, (b) Instantaneous reciprocal ellipticity computed with the three-components of the signal.

In Figure 6 (top), the Stockwell transforms of the *NEV* components filtered with the NIP criterion to extract retrograde Rayleigh waves is shown. If compared with the Stockwell transforms of the *NEV* components filtered with the instantaneous reciprocal ellipticity shown in Figure 5, it can be observed that the area before 30 seconds between 0.2 and 0.4 Hz is excluded by the NIP criterion. This is because that area corresponds to prograde motion as it can be seen in Figure 5(a). The extracted waves, obtained by inverting the Stockwell transforms are shown in Figure 6 (bottom). We can observe they include only the lower frequencies.

Now using the extracted waves of Figure 6 (bottom) we compute the angle of propagation of the Rayleigh waves using Eq. (2): $\theta$=257.99 deg. The angle is slightly different from that computed in the previous section. Next, the horizontal filtered components are rotated in the radial and transverse direction using this angle. The results are shown in Figure 7 (left). The waves in the Radial and Vertical components have an appearance strongly suggesting they are Rayleigh waves. We compare these two components in Figure 7 (left). Clearly, the Radial component and the Vertical component shifted 90 degrees are in-phase, confirming the waves we extracted are Rayleigh waves. The correlation coefficient between them is now 0.9018, showing in a quantitative manner the better results obtained when the NIP criterion is used.

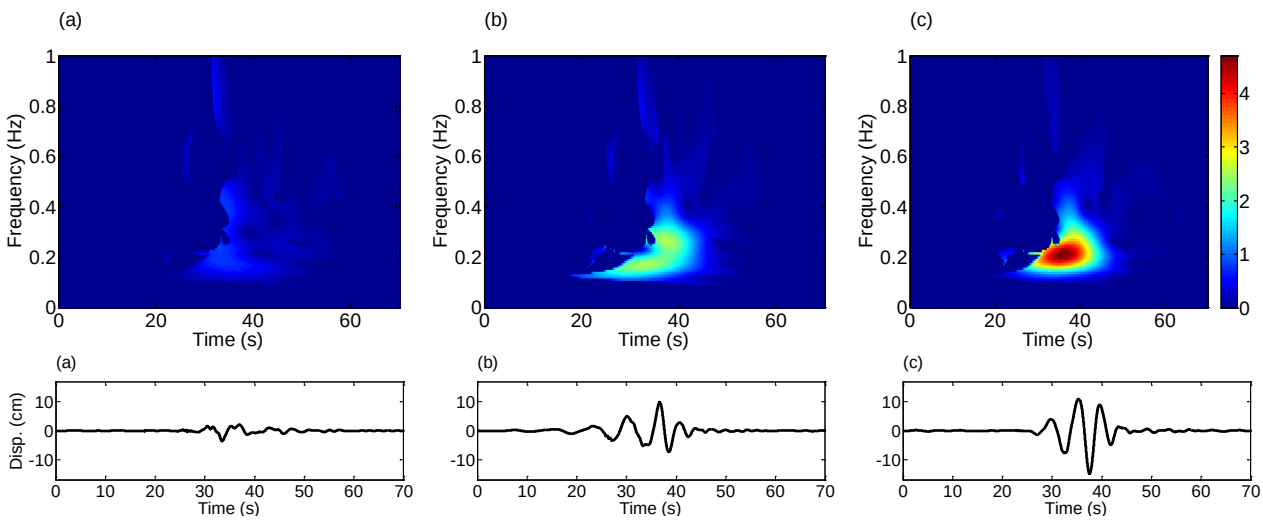


Figure 6. Filtered Stockwell transforms of the NEV displacements using the NIP criterion (top) and filtered NEV components of the displacement history: (a) North component, (b) East component, (c) Vertical component.

Finally, in Figure 7 (right), we show the comparison between the Radial component and the shifted Vertical component of the extracted prograde waves. The two components are in phase, and thus the extracted waves are confirmed as Rayleigh waves. We can also observe that the frequencies of these prograde waves are higher than those of the retrograde waves. The angle computed with the *NE* components of the prograde waves is: $\theta$=282.70 deg. This value is significantly different from the one obtained for the retrograde waves, indicating that most likely they are generated by different mechanisms. Such conclusion illustrates the advantage of using the NIP to extract Rayleigh waves.

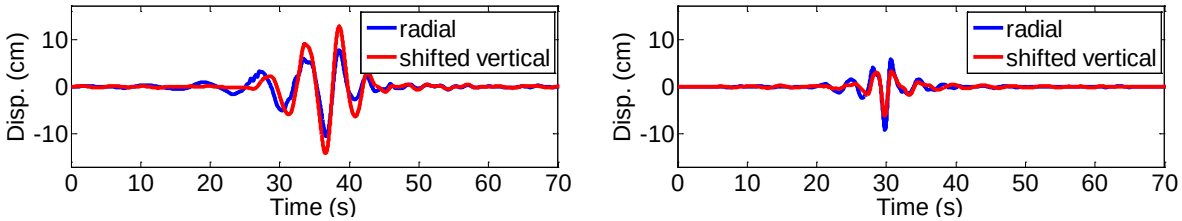


Figure 7. Comparison between the filtered Radial and shifted Vertical components of displacement history using the NIP criterion (left) and for prograde waves only (right). The Vertical component is shifted 90 degrees.

In Figure 8 (2nd column), we can observe the Radial and Shifted Vertical components of extracted Rayleigh waves using the *NIP* criterion for retrograde waves at different stations of the WCP plain in Taiwan. The results show the method is stable when applied to this dataset of seismic time histories. The same can be concluded for the extraction of the prograde waves, displayed in Figure 8 (3rd column).

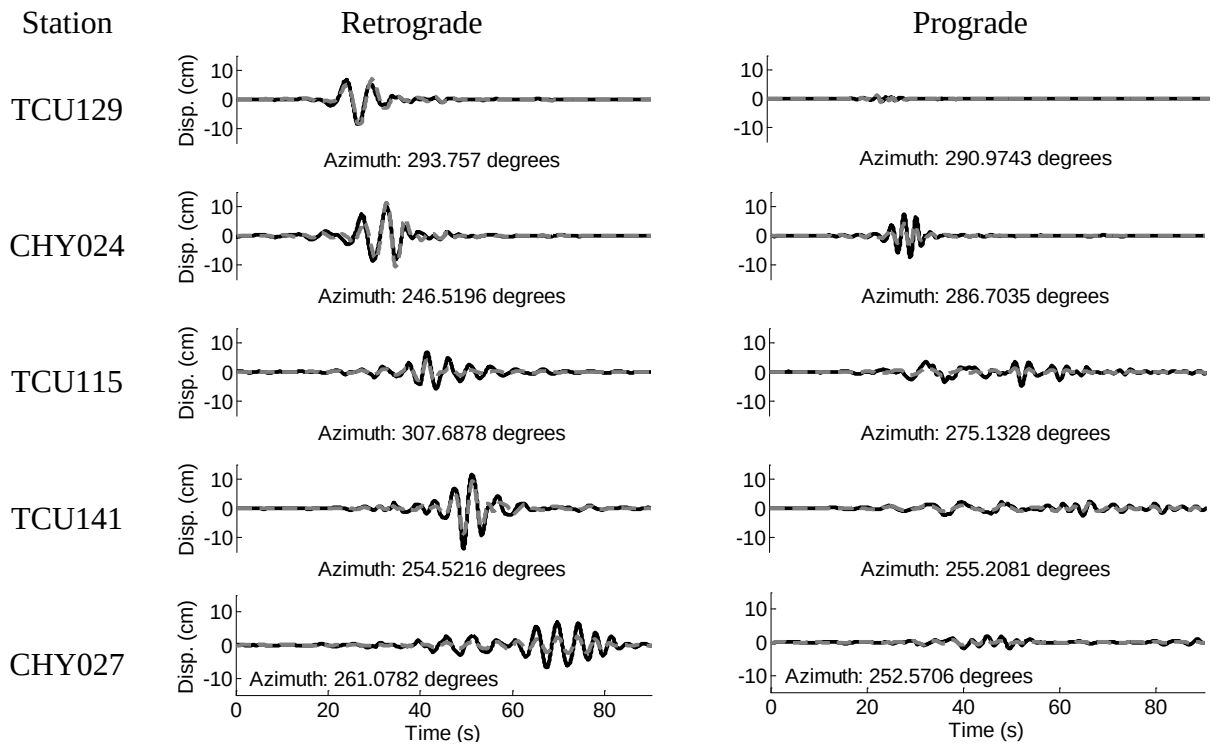


Figure 8. Radial component (black solid line) and shifted vertical component (gray dashed line) of extracted retrograde (2nd column) and prograde (3rd column) Rayleigh waves at different stations at the WCP plain in Taiwan.

## Conclusions

We have developed a method to extract Rayleigh waves from three-component displacement histories. The proposed method, based on the Normalized Inner Product, does no longer require *a priori* estimations of the frequency range and direction of propagation of the surface waves. We have shown that the method gives better results when compared with recently proposed methods to extract surface waves based on the Instantaneous Reciprocal Ellipticity, since the algorithm proposed herein distinguishes between prograde and retrograde particle motion. Therefore waves generated by different mechanisms will be more easily identified. Besides, numerical examples show that the Normalized Inner Product is more stable over the time-frequency domain than the Normalized Reciprocal Ellipticity, reducing the numerical artifacts due to filtering. The extracted waves can then be used to study the physics of surface wave generation and propagation in sedimentary basins, enhancing our capabilities to assess the seismic hazard at a site.

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