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Influence of Peak Factors on Random Vibration Theory Based Site Response Analysis

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ABSTRACT

Site response analysis based on the random vibration theory (RVT) approach often is used to estimate site amplification in probabilistic seismic hazard analysis for nuclear facilities. However, studies have shown that RVT analysis provides a systematic over-prediction of site amplification at the natural site frequencies when compared with the results obtained from a suite of input time series. A critical part of the RVT approach is the peak factor, which represents the ratio of the peak to *rms* value of a signal. This study investigates the influence of different peak factor models on the site amplification predicted by RVT analysis. Comparisons show that the use of the Vanmarcke (1975) peak factor model, rather than the commonly used Cartwright and Longuet-Higgins (1956) peak factor model, provides RVT estimates of site amplification similar to those from time series analysis. However, this agreement only occurs when the first mode site frequency is larger than the corner frequency of the input motion. For other cases, further refinements to the Vanmarcke (1975) peak factor model are necessary.

Introduction

Local site conditions affect the characteristics of earthquake ground shaking, and this effect is quantified though site response analysis. Often, 1-dimensional (1-D) site response analysis is used to model shear wave propagation from the base rock through a soil column to the ground surface. 1-D site response analysis typically requires a rock acceleration-time history as input and provides as output a surface acceleration-time history with an associated surface acceleration response spectrum and period-dependent spectral amplification factors (i.e., the ratio of surface to rock spectral acceleration at each period). A suite of input time histories is used in site response analysis to generate a statistically stable estimate of the site response. Using a suite of time series may involve heavy computations for probabilistic analyses and it may be difficult to identify a sufficient number of appropriate input motions in areas with a paucity of earthquake recordings.

An alternative is the Random Vibration Theory (RVT) approach to equivalent linear site response analysis (e.g., Silva et al. 1997, Rathje and Ozbey 2006, Kottke and Rathje 2013). This approach uses the same wave propagation theory as analyses that use input time series, but does not require input time histories. RVT based site response analysis combines Parseval's theorem and extreme value statistics to predict the peak time domain values of motion from the Fourier

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Amplitude Spectrum of motion. Parseval's theorem is used to compute the root-mean-square (*rms*) acceleration and extreme value statistics is used to estimate the peak-to-*rms* ratio, called the peak factor. The product of these values provides the peak acceleration or spectral acceleration of the motion. Researchers have validated the application of RVT in site response analysis (Silva et al. 1997, Rathje and Ozbey 2006), but more recent comparisons between time series (TS) and RVT equivalent linear site response analysis have shown that RVT analysis may predict amplification at the natural frequencies of a site 20 to 50% larger than TS analysis (Kottke and Rathje 2013).

This paper investigates the influence of different statistical models for the peak factor on the site amplification predicted by RVT-based site response analysis. The influence is studied through a comparison between RVT and TS results for different peak factor models. Earthquakes of different magnitudes and sites with different natural frequencies are used to better understand how earthquake magnitude and site frequency influence the results.

RVT Procedure and Peak Factor Models

RVT based site response analysis uses a Fourier Amplitude Spectrum (FAS) and a duration of ground motion as input. The basis of the RVT approach is that the peak acceleration of a signal is the product of the *rms* value and an estimated peak factor (*pf*). The *rms* value is estimated using Parseval's theorem (Boore and Joyner 1984):

$$a_{rms} = \sqrt{\frac{1}{D_{rms}} \int_0^{D_{rms}} |a(t)|^2 dt} = \sqrt{\frac{2}{D_{rms}} \int_0^{\infty} |A(f)|^2 df} = \sqrt{\frac{m_0}{D_{rms}}} \tag{1}$$

where D_{rms} is the *rms* duration obtained from the ground motion duration, D_{gm} (e.g., Boore and Joyner 1984), $A(f)$ is the Fourier amplitude at frequency f, and m_0 is the zero-order spectral moment of the FAS.

The expected value of the peak factor is obtained from statistical models derived from extreme value statistics. Many models of the statistical distribution of the peak factor have been proposed. Cartwright and Longuet-Higgins (CL) (1956) proposed a model for the statistical distribution of the peak factor for a stationary Gaussian process with zero mean and independent peaks. The equation for the expected value of CL *pf* is (Cartwright and Longuet-Higgins 1956, Boore 2003):

$$\overline{pf} = \sqrt{2} \int_0^{\infty} \left\{ 1 - \left[1 - \xi e^{-\eta^2} \right]^{N_e} \right\} d\eta \tag{2}$$

$$N_e = \frac{1}{\pi} \sqrt{\frac{m_4}{m_2}} D_{gm} \tag{3}$$

where ξ is a bandwidth factor, N_e is the number of extrema, and m_i is the i-th order spectral moment of the FAS (Boore 2003). The CL *pf* has been incorporated into most RVT procedures in engineering seismology (e.g., Boore 2003) and site response (Kottke and Rathje 2008), but it fails to take into account the fact that the peaks of a narrow-band process are not independent and often occur in clumps (Vanmarcke 1975).

Vanmarcke (1975) developed a model for the statistical distribution of the peak factor based on a first-passage problem. This approach allows for the clumping of peaks to be taken into account. The complementary cumulative distribution function of the peak factor from Vanmarcke (1975) is written as:

$$P(pf > b) = [1 - \exp(-b^2/2)] \cdot \exp[(-N_z\left(1 - e^{-\delta^{1.2} b\sqrt{\pi/2}}\right)/\left(e^{b^2/2} - 1\right)] \quad (4)$$

$$N_z = \frac{1}{\pi}\sqrt{\frac{m_2}{m_0}} D_{gm} \quad (5)$$

where δ is a bandwidth factor, which is different than the bandwidth factor used by CL, and N_z is the number of zero-crossings. The expected value of the Vanmarcke *pf* can be obtained by differentiating Equation 4 to obtain the probability density function (pdf) and integrating the pdf over all values of *pf*. The Vanmarcke and CL bandwidth factors are defined as (Cartwright and Longuet-Higgins 1956, Boore 2003, Vanmarcke 1970):

$$\xi = \frac{m_2}{\sqrt{m_0 m_4}} = \frac{N_z}{N_e} \quad (6)$$

$$\delta = \sqrt{1 - \frac{{m_1}^2}{m_0 m_2}} \quad (7)$$

where m_i is the i-th order spectral moment (Boore 2003). Both bandwidth factors range from 0 to 1, yet δ approaches 0 for a narrow-band process while ξ approaches 1.0 for a narrow-band process. When computing a response spectrum in RVT site response analysis, peak factor models should be used with appropriate D_{rms} models. The D_{rms} model accounts for the fact that the duration of the oscillator response can be longer than the ground motion duration (Boore and Joyner, 1984). Available D_{rms} models have been developed empirically by comparing RVT response spectra and response spectra of stochastically generated acceleration-time histories. As a result, these models are linked to the peak factor model used in the RVT analysis. Most of the available D_{rms} models (e.g., Boore and Thompson 2012, BT12) were developed using the CL *pf*. However, Boore and Thompson (2015, BT15) recently developed a D_{rms} model using the Vanmarcke *pf*.

Because the bandwidth factors ξ and δ are defined differently and cannot be uniquely related, it is not possible to systematically compare the CL and Vanmarcke peak factors without prescribing the frequency content of the signal. Towards this end, the FAS for an earthquake motion with a *M* 6.5 and *R* 20 km is defined using a theoretical seismological model (Brune 1970, 1971) for conditions in Central and Eastern North America (CENA). The FAS and its oscillator response for an oscillator frequency of 5 Hz and 5% damping are shown in Figure 1a. For the rock FAS ξ and δ are 0.475 and 0.704, respectively, indicating a relatively broad-band motion. For the oscillator FAS, ξ becomes larger (ξ = 0.854) and δ smaller (δ = 0.242), indicating that the oscillator response is a more narrow-band process than the rock motion. Figure 1b shows the expected values of the CL *pf* and Vanmarcke *pf* for both motions as a function of N_z. For the broad-band rock motion the CL and Vanmarcke *pf* yield similar values, yet for the narrow-band oscillator motion the CL *pf* is generally about 10% larger than the Vanmarcke *pf*. This comparison shows that for narrow-band processes, such as the oscillator response of earthquake motions represented by a response spectrum, the Vanmarcke (1975) predicts a smaller peak factor than CL, because it accounts for the dependence/clumping between peaks. This effect will be magnified for surface ground motions, and thus using the Vanmarcke *pf* will influence predicted site amplification from RVT.

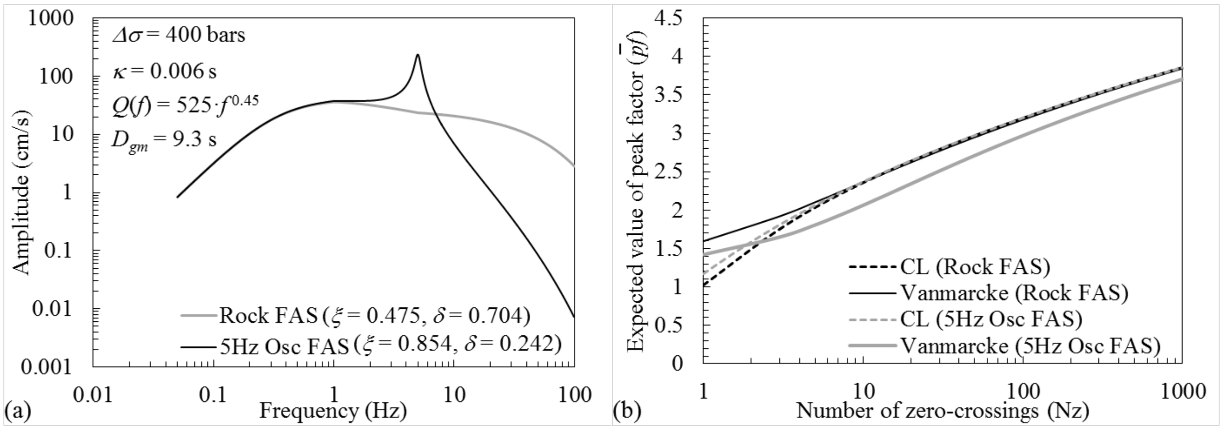

Figure 1. Comparison of CL and Vanmarcke peak factors for a M = 6.5, R = 20 km earthquake

Analyses Performed

Linear site response analyses using both the TS and RVT approach are conducted for a range of site conditions and earthquake magnitudes. Hypothetical sites similar to those used by Kottke and Rathje (2013) are used. Each site is composed of a single layer soil deposit and an underlying rock half-space. The shear wave velocity and the unit weight for the soil layer are 400 m/s and 18 kN/m^3, and for the rock layer they are 3,000 m/s and 22 kN/m^3. The damping ratio for both layers is 1%. Sites 100 m and 316 m thick are used to show the influence of the natural site frequency. The corresponding first mode site frequencies are 1.0 Hz and 0.32 Hz, respectively.

The input motions for both the RVT and TS site response analyses are computed using the program SMSIM (Boore 2005). A fixed distance of 20 km and three magnitudes, M 5, M 6.5, and M 8, are considered. The input motions for RVT analyses are Fourier amplitude spectra, generated using a single corner frequency source spectrum and seismological source theory (Boore 2003), and ground motion duration, derived from Boore and Thompson (2015). The important seismological parameters describing the shape of the FAS include the stress drop ($\Delta\sigma$) and site diminution (κ), values of which are shown in Figure 2. These values represent conditions for CENA based on the most recent recommendations of Boore (2015). For each magnitude, the input for the TS analyses is a suite of 100 time series generated using stochastic simulation (Boore 1983) based on the same Brune FAS as the RVT calculations. As a result, the arithmetic mean of the FAS of the time series matches the FAS used in the RVT analysis. The Brune FAS and the D_{gm} used for all three magnitude events are shown in Figure 2. The corner frequencies are 1.8 Hz, 0.33 Hz, and 0.06 Hz for the M 5.0, 6.5, 8.0 events, respectively.

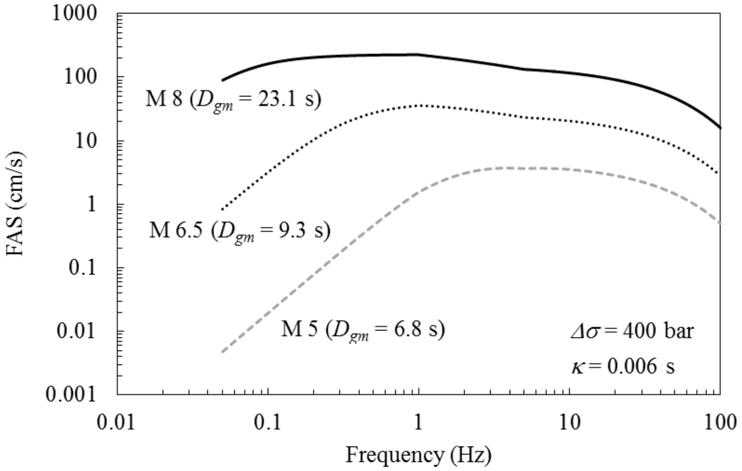

Figure 2. Input FAS and ground motion durations for M 5, M 6.5, and M 8 earthquakes

Site Amplification Results

The site response results are shown in terms of the amplification factor (*AF*), defined as the ratio of the surface spectral acceleration to the rock spectral acceleration. For the TS analyses the median of the 100 input time series is shown. Figures 3 shows the TS and RVT site amplification for the two sites subjected to the M 6.5 event. For each site and each motion, the *AF* peaks at the natural frequencies of the site, but the RVT results using the CL *pf* are generally larger than the TS results at the modal frequencies. The RVT analyses using the Vanmarcke *pf* agree well with the TS results, but the agreement is site dependent. For the 100-m thick site, the *AF* using the Vanmarcke *pf* agrees very well with the TS result at the first mode. At the second mode, the result from the Vanmarcke *pf* is larger than the TS result, but slightly smaller than the results from the CL *pf*. At higher modes, very little or even no improvement is observed when using the Vanmarcke *pf*, although the difference between RVT and TS is relatively small here. For the 316-m thick site, the *AF* using the Vanmarcke *pf* shows improvement by about 50% at the first and second modes relative to the CL *pf*, but there is little improvement at higher modes. Although the level of improvement for the first two modes are different for the two sites, the results in Figure 3 show that the Vanmarcke *pf* is effective in reducing the over-prediction in site amplification from RVT analysis. This improvement is due to the fact that Vanmarcke *pf* considers the narrow-band characteristic of the oscillator response at the modes. At the first few modes, the bandwidth factor δ is as low as 0.1 (e.g., Figures 3c and d) and the Vanmarcke *pf* is significantly smaller than the CL *pf* at these frequencies (Figures 3e and f), thus reducing the site amplification in the RVT analysis. At the higher modes, the motion is no longer narrow-band, as indicated by the bandwidth factors, and the Vanmarcke and CL *pf* provide the same result.

To further investigate the improvement in RVT site response analysis when using the Vanmarcke *pf*, the site amplification for the H = 316 m site is calculated for earthquake magnitudes of 5.0, 6.5, and 8.0. The site response results shown in Figure 4 demonstrate that earthquake magnitude significantly influences the differences between TS and RVT site response analyses. For the M 5.0 event (Figure 4a), there is a significant difference between the RVT and TS results, with the RVT analysis predicting an *AF* almost 70% larger than TS analysis at the

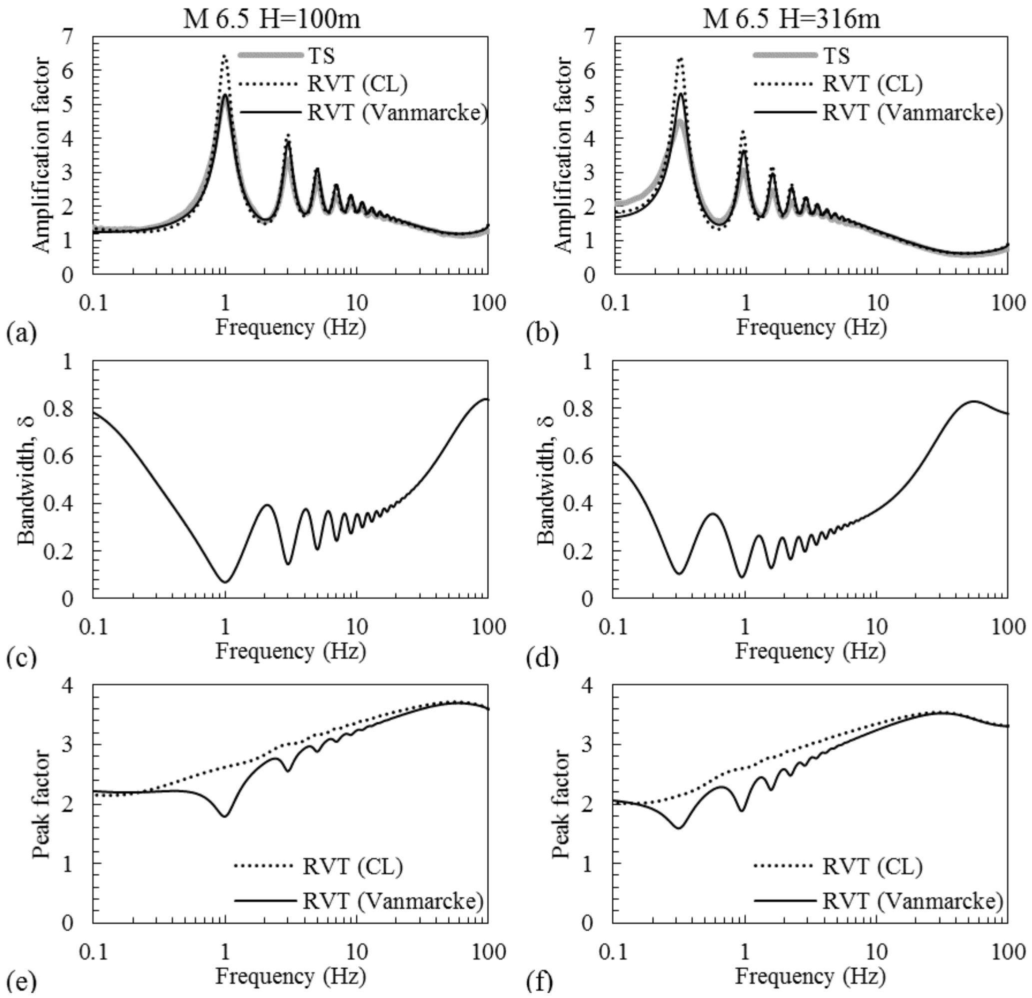

Figure 3. RVT and TS site response results for 100 m and 316 m sites and M 6.5 earthquake

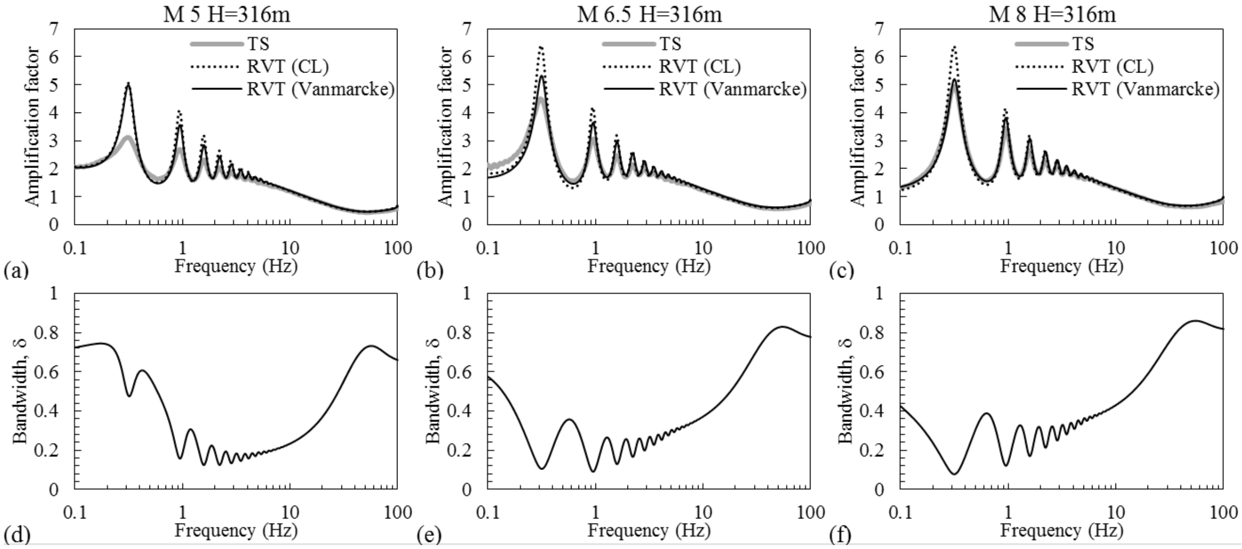

Figure 4. RVT and TS site response results for 316 m sites and M 5, 6.5, and 8 earthquakes

first mode. For the M 6.5 event (Figure 4b), the RVT analysis is about 15% larger than the TS analysis at the first mode, and for the M 8.0 event (Figure 4c) there is very good agreement between the RVT and TS analyses. In Figure 4 the RVT amplification values are very similar for the three different magnitude events (AF ~ 5.0 at the first mode), but the TS amplification values increase with increasing magnitude (AF ~ 3.0 at the first mode for M 5.0, AF ~ 5.0 at the first mode for M 8.0).

Earthquake magnitude modifies the input motion in two important ways: the low frequency content, as characterized by the corner frequency, and the ground motion duration. Considering the response of several sites with depths between 32 m to 316 m, it appears that RVT becomes inaccurate when the first mode site frequency is smaller than the corner frequency of the input motion (i.e., deeper sites subjected to smaller magnitude earthquakes, Figures 3 and 4). To isolate the effect of duration, analyses were performed with a M 5.0 spectral shape and a longer duration of 23.1 s (Figure 5). The results in Figure 5 show that the longer duration increases the AF predicted by TS analysis, but does not affect the RVT amplification. For short duration motions, the site amplification is smaller for the TS analysis because the forcing function (i.e., input motion) ends before the dynamic response of the soil is fully realized. However, the change in duration does not impact the RVT results appreciably. The amplification from RVT is not noticeably influenced by duration because: (1) RVT assumes a stationary signal and thus a steady state response, no matter the duration, when it computes a_{rms}, and (2) the same duration is used to compute the rock response spectrum and surface response spectrum.

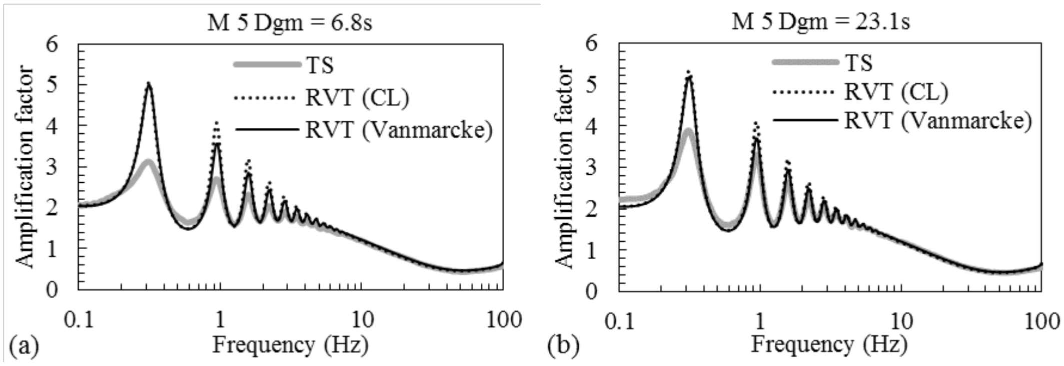

Figure 5. Effect of duration on site amplification for a M 5 earthquake

Conclusions

Accurately predicting earthquake motions and earthquake response using RVT requires an accurate model for the peak factor. The Vanmarcke (1975) peak factor model is superior to the Cartwright and Longuet-Higgins (1956) peak factor model because it considers the dependence between peaks in a signal, and thus more accurately captures the peak response of narrow-band processes. Comparisons between TS and RVT site response analyses confirm that the Vanmarcke (1975) peak factor model provides more accurate estimates of site amplification than the Cartwright and Longuet-Higgins (1956) peak factor model. However, the RVT results using the Vanmarcke (1975) peak factor are accurate only for some cases; specifically, the approach is accurate when the first mode site frequency is larger than the corner frequency of the input motion. For cases in which the first mode site frequency is smaller than the corner frequency of the input motion (e.g., deeper sites and smaller magnitude earthquakes), further refinements to the Vanmarcke (1975) peak factor model are necessary.

The work presented here involved only linear elastic site response analyses of sites with uniform shear wave velocity. Further work is needed to investigate sites with velocities that vary with depth and to analyze sites as equivalent-linear. When equivalent-linear analysis is performed, RVT is also used to predict the peak shear strain to define the strain-compatible properties, and thus the accuracy of the RVT strain computation requires investigation.

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