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Comparisons of One-Dimensional Site Response Analysis and Borehole Array Observations: Quantification of Bias and Variability

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ABSTRACT

The accuracy of one-dimensional site response methods are investigated by analyzing data from instrumented borehole arrays. Eleven instrumented vertical arrays are investigated using over 650 recorded ground motions that span a range of shaking intensities and induced strain levels. Predictions of site amplification from one-dimensional equivalent linear (EQL) analysis, equivalent linear analysis with frequency-dependent soil properties (EQL-FD), and fully nonlinear analysis (NL) are compared with those from the surface and borehole recordings. These comparisons show that, on average, all three of the site response methods predict site amplification within +/-20% of observed values what the induced peak shear strain is less than about 0.2%. At induced peak shear strains larger than 0.2%, the EQL and NL analyses under-predict site amplification and EQL-FD analyses over-predict site amplification at periods less than about 0.4 s. The under-prediction for the EQL and NL techniques may be as large as 65% to 75%, and the over-prediction for the EQL-FD technique may be as large as 75%. The variability in the site amplification residuals is separated into inter-site and intra-site components. At small strains, the inter-site variability and intra-site variability are similar in magnitude, but at larger strains the inter-site variability dominates.

Introduction

Numerical site response analysis commonly is performed to quantify the effects of local soil conditions on the characteristics of earthquake shaking at a site. In cases where topographic and basin effects are minimal, one-dimensional (1-D) site response analysis is considered adequate to represent the wave propagation conditions (Kramer 1996). One-dimensional site response analyses involve the propagation of shear waves from the base rock to the ground surface through a model of the soil layers. Different 1-D approaches model the nonlinear response of the soil using different techniques. Fully nonlinear (NL) analysis tracks the nonlinear stress-strain behavior in the time domain during earthquakes shaking. The equivalent-linear (EQL) approach approximates the nonlinear aspects of the soil response within a linear-elastic framework, iteratively adjusting the linear elastic soil properties to be consistent with an effective level of shear strain induced in the soil. Equivalent-linear analysis with frequency dependent soil properties (EQL-FD) is an equivalent-linear approach that considers the frequency-dependence of the shear strain to define strain-compatible properties at each frequency. This approach has been proposed to address the inability of EQL analysis to accurately predict site amplification at

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high frequencies when the induced strains are large (e.g. Sugito et al. 1994, Assimaki and Kausel, 2002; Yoshida et al., 2002).

The main objective of this study is to evaluate the EQL, EQL-FD, and NL approaches to site response analysis using recordings from borehole arrays. The site amplification computed from each numerical technique is compared with observations obtained from the borehole arrays. The ability of the different numerical techniques to accurately predict the ground response under low to moderate to strong shaking is investigated, and the variability associated with the theoretical models is quantified. This paper extends the work presented in Zalachoris and Rathje (2015).

Site and Ground Motion Selection

The Kiban-Kyoshin (Kik-Net) database in Japan was the major source for borehole array data. The selection of sites was based on the shear wave velocity characteristics and the availability of ground motions at each site. Sites were selected with V_{S30} (i.e., time-averaged shear wave velocity over the top 30 m) less than about 400 m/s, because they will exhibit stronger nonlinearity, and with the availability of high intensity recordings. Using these criteria, nine KiK-Net sites were identified. To capture the response of softer sites, the La Cienega and Lotung downhole array sites also were analyzed. A total of 650 recordings were used.

The shear wave velocity profiles of the eleven selected sites are shown in Figure 1 along with location of the borehole sensor. The V_{S30} for the 11 selected sites ranges from 184 m/s to 371 m/s, while the depth to the borehole sensor varies from “shallow” (≤ 100 m) to “deep” (> 200 m). The recordings at the borehole sensor were used as input into the site response analyses. The EQL and EQL-FD analyses were performed in the frequency domain assuming a “within” wave field at the base, which essentially models a rigid base. The program Strata (Kottke and Rathje, 2008) was used for these analyses. The NL analyses were conducted assuming a perfectly rigid base. These analyses were performed using the program DeepSoil (Hashash, 2012). The layering at each site was defined based on the measured shear wave velocities and the nonlinear modulus reduction and damping curves were assigned from the Darendeli (2001) model using assumed values of plasticity index (PI) and depth-dependent confining pressure. The assigned values of small-strain damping (D_{min}) were scaled up from those predicted by Darendeli (2001) to account for other mechanisms of energy dissipation (e.g., wave scattering) and to better match the recorded amplification at small strains. (Zalachoris, 2014; Zalachoris and Rathje, 2015). For the sites analyzed, the 1D approach captured well the amplification at seven sites. For the other sites, the 1D approach over-estimated the first mode response, likely due to the large depth of the base sensor (Zalachoris and Rathje, 2015).

Assessment of 1-D Site Response Methods

The assessment of the site response methods is based on the period-dependent amplification (AF) represented by the ratio of the surface acceleration response spectrum divided by the input acceleration response spectrum. This study focuses on amplification in terms of acceleration response spectra because of their significant use in engineering applications. Amplification is investigated over the period range of 0.05 s to 2.0 s because over this period range the signal-to-noise ratio was greater than 5 for the vast majority of the recordings.

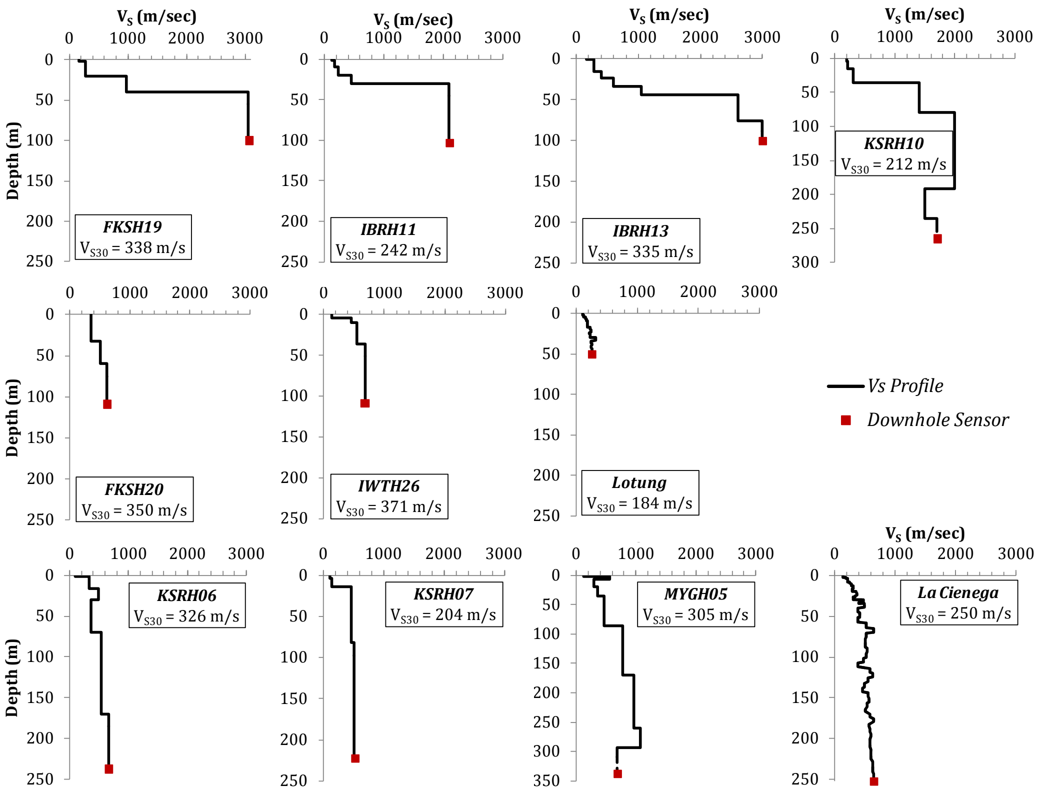


Figure 1. Shear wave velocity profiles of the borehole arrays used in this study.

Quantification of Bias

A framework developed by Abrahamson *et al.* (1990) for the quantification of the goodness-of-fit between ground motion simulations and observations is used. Abrahamson *et al.* (1990) quantified the misfit in terms of the difference between the natural log of the response spectral acceleration values. In this study, we quantify the misfit (or bias) as the difference between the natural log of the observed and calculated amplification factors (AF) at each period. A similar framework was used by Kaklamanos et al. (2013). For the i^{th} site and j^{th} recording, the residual (y) at period T given the observed amplification factor (AF^{obs}) and the calculated amplification factor (AF^{calc}) is given by:

$$y_{i,j}(T) = ln\ AF^{obs}_{i,j}(T) - ln\ AF^{calc}_{i,j}(T) \qquad (1)$$

A positive residual represents under-prediction by the model, while a negative residual represents over-prediction by the model. The calculated residuals are classified according to the magnitude of the induced shear strains (γ_{max}), as computed by the site response analysis. The maximum shear strain is the parameter most influencing the accuracy of the predictions because it directly influences the shear modulus and damping values used in the site response computations.

Following the framework in Equation 1, the performance of the site response techniques across all 11 sites is assessed by aggregating the computed amplification residuals as a function of spectral period (T) and maximum calculated shear strain (γ_{max}). To simultaneously present the results as a function of both T and γ_{max}, contour plots of the computed mean residuals are developed for all three techniques and plotted against the corresponding values of T and γ_{max} (Figure 2). The mean prediction residuals are colored based on their sign with strong positive residuals (i.e., under-prediction) shown in red and strong negative residuals (i.e., over-prediction) shown in blue. A range of "acceptable" prediction residuals is colored in white/grey. This "acceptable" range of prediction residuals is defined as ±0.2, which corresponds to the computed AF falling within approximately ±20 % of the observed AF.

The results in Figure 2 show that the three techniques generally predict very similar, small residuals (i.e., ±0.2) at strains less than 0.2%. There are some residuals that fall outside the acceptable level at small strains, but these results are driven by the data from only two sites (FKSH19 and IBRH11). At strains larger than about 0.2%, the EQL and NL techniques under-predict the observed response at periods less than 0.3 to 0.4 s, and the under-prediction becomes more significant with increasing shear strain. In fact, the predicted AF may be as small as one-third of the observed value (residual close to 1.0) at strains close to or greater than 1.0 %. For the EQL technique, the under-prediction is a result of over-damping of the high frequencies due to the large peak strain used for the selection of strain-compatible soil properties. Although NL analysis models the fully nonlinear stress-strain response of the soil and does not use strain-compatible properties, the results in Figure 2 show that NL analysis is still prone to the same under-prediction of short periods at large strains.

For the EQL-FD technique, amplification residuals become negative at strains greater than about 0.01 % and periods smaller than 0.3 to 0.4 s. Between 0.01% and 0.2% the EQL-FD method over-predicts the response by about 50% (residual = -0.4) and at strains greater than 0.2% the over-prediction is as large as 75% (residual = -0.55). The over-prediction at short periods is caused by the fact that the strains at short periods are very small and as a result the EQL-FD method utilizes small damping values at these periods, which leads to a larger response.

Quantification of Variability

The results presented above focused on the differences, on average, between the observed and predicted responses at 11 downhole array sites. However, considering all of the recorded data, there is considerable variability between the observations and predictions. Investigating this variability provides additional insights into the performance of site response analysis.

The different components of variability can be evaluated using methodologies proposed by several researchers in the context of either ground motion prediction equations (e.g., Al Atik et al., 2010; Lin et al., 2011; Rodriguez-Marek et al., 2011) or site response analysis (e.g., Kaklamanos et al., 2013). Assuming that the observation residuals at a specific frequency, $y_{i,j}$, represent a normally distributed random variable with mean μ_y and standard deviation σ_y, we can separate the residuals into different components, as (Kaklamanos et al. 2013):

$$y_{i,j} = \alpha + \eta_{s,i} + \varepsilon_{i,j} \qquad (2)$$

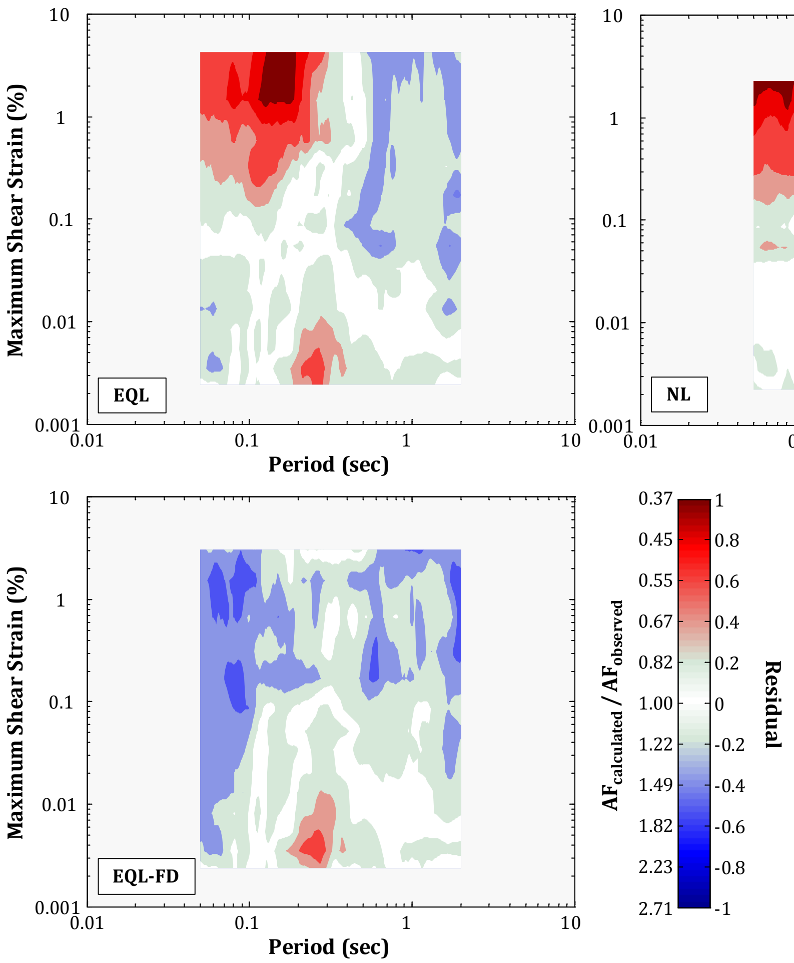


Figure 2. Computed mean prediction AF residuals as a function of period and γ_{max} for all sites and site response methods.

where α is the "fixed effect", $\eta_{s,i}$ is the inter-site residual, and $\varepsilon_{i,j}$ is the intra-site residual. The "fixed effect", α, represents the mean bias across all sites and recordings. The inter-site residual, $\eta_{s,i}$, quantifies the "site-specific" bias and is assumed to be a normally distributed random variable with, ideally, zero mean ($\mu_{\eta s}= 0$) and standard deviation of τ_s. For the i^{th} site, $\eta_{s,i}$ represents the misfit between the mean residual of the site and the "fixed effect", α. The intra-site residual, $\varepsilon_{i,j}$, expresses the "within-site" variability and is a normally distributed random variable with zero mean ($\mu_\varepsilon= 0$) and standard deviation of σ. $\varepsilon_{i,j}$ represents the difference between a single observation, $y_{i,j}$, and the site-corrected mean residual ($\alpha + \eta_{s,i}$). Considering Equation 2, the mean (μ_y) and the total standard deviation (σ_y) of the observation residuals can be split into three components, namely: the fixed effect, α; the inter-site standard deviation, τ_s; and the intra-site standard deviation, σ, given by:

$$\mu_y = \alpha \tag{3}$$

$$\sigma_y = \sqrt{\tau_s^2 + \sigma^2} \tag{4}$$

By grouping the results based on the computed γ_{max}, we are able to quantify the variability of the residuals for the EQL, EQL-FD and NL methods at different strain levels at each period. Three bins of γ_{max} values are used; $\gamma_{max} \leq 0.01\%$, $0.01\% < \gamma_{max} \leq 0.10\%$, and $\gamma_{max} > 0.10\%$. The number of motions within each strain bin varied substantially. For example, for the EQL analyses, 324 motions are in the $\gamma_{max} \leq 0.01\%$ bin and 279 motions are in the $0.01\% < \gamma_{max} \leq 0.10\%$ bin, but only 55 motions are in the $\gamma_{max} > 0.10\%$ bin.

Figure 3 illustrates the mean residuals for the EQL, EQL-FD and NL methods for the three strain bins, and Figure 4 presents the corresponding standard deviations. The results in Figure 3 are a different representation of the results shown in Figure 2. They show that at $\gamma_{max} \leq 0.01\%$ all of the site response methodologies yield almost identical mean prediction residuals (μ_y) across all spectral periods and are generally within the range of ± 0.2 at periods less than 1.0 s. As the calculated shear strains increase, deviations in μ_y between the different models are evident. At $0.01\% < \gamma_{max} \leq 0.10\%$, the methods start to deviate from one another but the mean residuals still stay with the range of ± 0.2. The most significant differences in the residuals from the different site response techniques can be seen at strains greater than 0.10%. Here, the EQL and NL residuals become increasingly positive for periods less than about 0.5 s and indicate an under-prediction in the response. The EQL-FD approach results in large negative residuals (i.e., over-amplification) at periods less than 0.2 s, but this over-prediction appears to be generally less than the under-prediction by the EQL and NL methods.

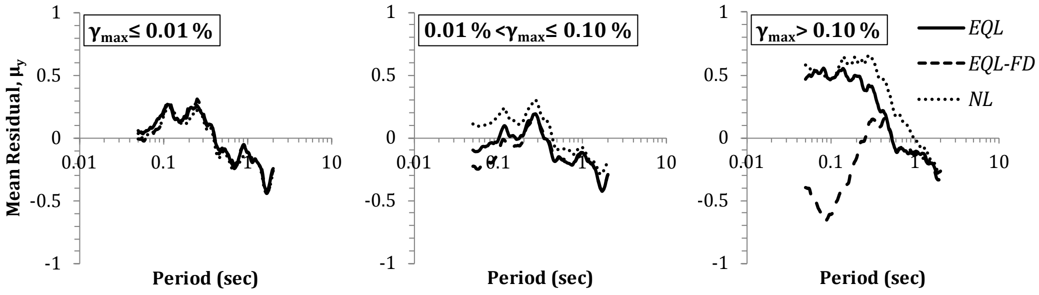


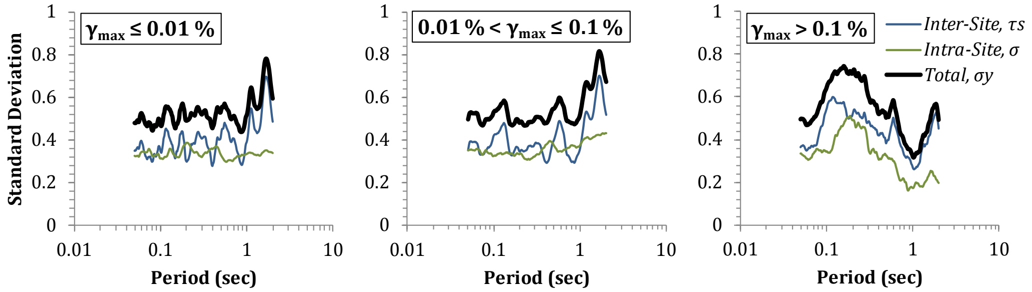
Figure 3. Mean residuals across different shear strain levels for the site response methods.

The computed standard deviations are shown in Figure 4 for the three strain bins and three site response techniques. For smaller strains (i.e., $\gamma_{max} \leq 0.01\%$ and 0.01 – 0.1% strain bins), the total standard deviation (σ_y) is approximately 0.45-0.5 at periods less than 1.0 s for the three methods, and the inter-site standard deviation (τ_s) and intra-site standard deviation (σ) contribute almost equally (~ 0.3 to 0.4) to the total standard deviation. The intra-site variability (σ) is essentially period independent and equal to about 0.35, with a slightly smaller value observed for the NL analysis. The inter-site standard deviation (τ_s), on the other hand, increases from about 0.4 for $T < 1.0$ s to almost 0.7 at $T > 1.0$ s. This result is caused by the fact that the "within" wavefield produces excessive first mode amplification at some sites. Because only some sites display this feature and it is only observed at long periods, the inter-site variability across all sites is increased at longer periods.

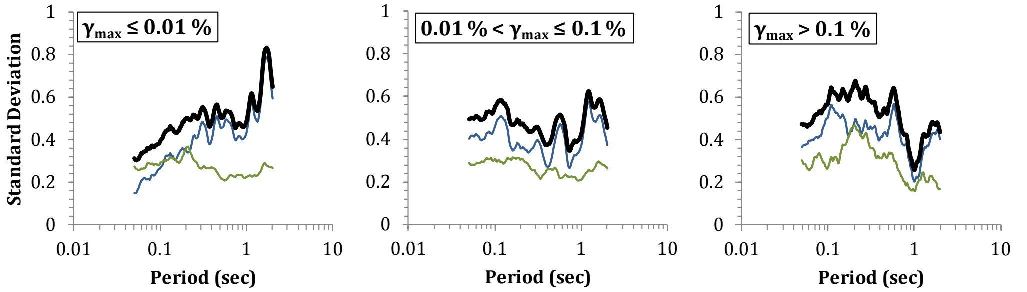
At larger strains ($\gamma_{max} > 0.10\%$), the total variability (σ_y) increases to about 0.65-0.8 at periods between 0.1 s and 0.4 s. Moreover, the peak in σ_y observed at smaller strains at $T > 1.0$ s is substantially decreased at larger strains. At $\gamma_{max} > 0.10\%$, the inter-site component of variability

(τ_s) dominates across all spectral periods, with the intra-site standard deviation being smaller, particularly for the EQL-FD approach. At these levels of shaking the calculated maximum strain predominantly influences the performance of the theoretical model, which may explain why the intra-site variability is smaller than the inter-site variability in this case. The larger inter-site variability (τ_s) can also be attributed to the fact that analyses resulting in large shear strains include fewer motions per site, which makes the estimates of the average inter-site residuals $(\eta_{s,i})$ less reliable and increases τ_s. The total standard deviation (σ_y) at small strain levels is very similar to that for the EQL and EQL-FD methods, further strengthening our conclusion that the overall variability in the predictive estimates is independent of the numerical scheme used to compute the site response.

(a) EQL



(b) NL



(c) EQL-FD

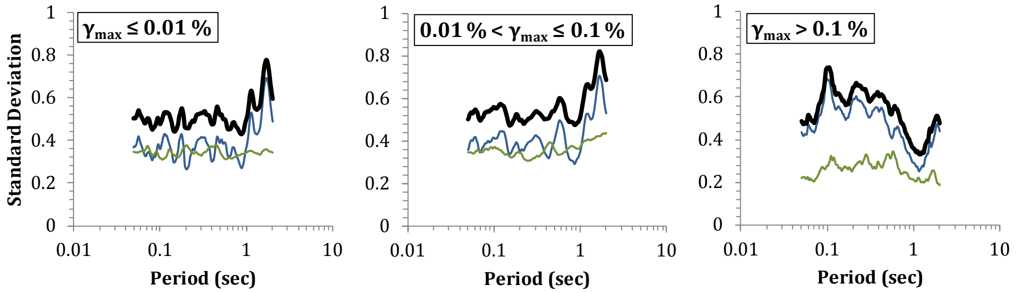


Figure 4. Standard deviations for amplification factors for all site response methods

Conclusions

Using recordings from borehole arrays, the accuracy of one-dimensional EQL, NL, and EQL-FD site response techniques was evaluated over a wide range of shaking intensities. The models were evaluated both in terms of the mean bias and its variability.

The mean bias showed that all three site response techniques (EQL, EQL-FD and NL) can accurately model site amplification at shear strains less than about 0.2%. However, for shear strains larger than 0.2%, the models become increasing inaccurate at periods less than about 0.4 s. Under these conditions EQL and NL analyses may under-predict site amplification by as much as 65% to 75% at short periods, while the EQL-FD technique may over-predict site amplification at short periods by as much as 75%. The variability associated with site amplification predicted by the EQL, NL and EQL-FD methods was evaluated in terms of the inter-site and intra-site contributions to the total variability. Across the three site response methods and for strains less than 0.1%, the total standard deviation is approximately 0.5 at periods less than 1.0 s and the inter-site and intra-site standard deviations contribute almost equally to the total standard deviation at these periods. At strains larger than 0.10%, the total variability increases to about 0.65-0.8 at periods less than 1.0 s. At these strain levels, the inter-site component of variability dominates across all spectral periods, with the intra-site standard deviation being smaller. However, with fewer data at large strains these variability estimates are more uncertain.

One limitation of this study is the use of assumed nonlinear modulus reduction and damping curves for the soil layers. This issue is most critical at large strains, such that different curves potentially could influence our conclusions related to model bias at large strains, and should be further investigated in future research.

Acknowledgments

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