

## Kinematic Pile Bending in Nonlinear Soils

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### ABSTRACT

In recent years the issue of kinematic interaction of deep foundations has received considerable attention both in the experimental and numerical research field. However, most of the numerical studies published in the literature implement simplified constitutive laws for the soil. A fundamental prerequisite of numerical modelling in earthquake geotechnical engineering is the ability to realistically reproduce soil response under cyclic loading. Several sophisticated constitutive models have been developed for such a scope, but their use in the practice is very limited due to high computation demand and difficulty in calibrating a lots of parameters generally required. To overcome some of these difficulties, the capability of a simplified kinematic hardening soil constitutive law has been investigated in order to analyze the seismic response of a single pile, embedded in a bi-layer soil deposit.

### Introduction

The seismic response of a building founded on piles is the result of a complex interaction among three different components: soil, foundation and superstructure, which mutually interacts each other. One of the main difficulties in modelling the observed response of a building on piled foundation is related to the nonlinear phenomena affecting the individual components (especially the soil) and their interfaces. Modelling the complete system by direct analysis with the real geometry (three-dimensional) and the proper constitutive laws for all materials is challenging and time consuming, so this approach is rarely used in the professional practice.

SSI problems are generally faced through the so-called substructure approach, which decouples the overall interaction into two distinct phenomena (kinematic and inertial interaction) and computation stages, later combined to provide the complete solution. In analyzing the kinematic interaction problem, the system consists of just two components: the soil and the foundation. The superstructure is assumed massless. At the end of this stage, the seismic motion transmitted to the superstructure FIM (Foundation Input Motion) and the (kinematic) stress states in the pile are computed.

In the second stage, the superstructure, excited by the FIM, is analyzed to get the actions transmitted to the foundation; the soil and the foundation now contribute to the global behaviour of the system through dynamic impedance functions. Subsequently, the foundation, loaded with the actions arising from the superstructure analysis, is analyzed. The stresses computed in the piles at this stage (inertial) will be added to those obtained from the kinematic analysis.

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## **Kinematic Interaction**

During an earthquake kinematic interaction originates in piles because they move differently from the surrounding soil. In particular, at the interface between two soil layers having different stiffness, the curvature of the soil  $(1/R)_s$  tends to infinity, due to the discontinuity of the shear deformation, while the curvature of the pile  $(1/R)_p$  has a finite value. The difference in pile-soil curvature induces kinematic bending moments in the pile, especially at the interface between soil layers having sharply different stiffness properties. Here the kinematic pile bending moments may even exceed those at the top of the pile if the top is restrained against rotation. Kinematic bending moments are affected by several factors such as: relative pile-soil stiffness, characteristics of the input motion, kinematic constraints at the base and cap of the pile, pile slenderness, thickness and geotechnical properties of the soil layers, presence of a fluid phase in the soil. Di Laora et al. (2012) show that the stiffness contrast between consecutive soil layers imposes a “constraint” along the pile, which can affect the sign and magnitude of the kinematic bending moment at the interface.

In the last decade kinematic interaction has been studied by several authors. As a result of this research effort, several simplified formulas are now available to predict pile kinematic bending moments at the interface between two soil layers or at the pile head. Most of these formulations have been obtained from extensive parameter studies by means of the BDWF (Beam on Dynamic Winkler Foundation) approach (Nikolaou et al., 2001; Mylonakis, 2001; Dezi et al., 2010; Sica et al., 2011 and 2013). Simplified formulas were also derived by means of parameter studies performed by continuum approaches (Maiorano et al., 2007; Di Laora et al., 2012).

The main shortcoming of all the above solutions lies in the constitutive model adopted for modelling soil response under cyclic loads. Linear (or equivalent linear) visco-elastic soil models are generally implemented. In literature only a few research works on seismic response of piles account for more refined soil constitutive laws (Kimura et al., 2000; Zhang et al., 2000; Zhang et al., 2002; Bentley et al., 2000; Martinelli, 2012).

## **Constitutive model adopted for the soil**

In the geotechnical field several constitutive models have been developed to simulate the main aspects of the complex soil response under cyclic and dynamic loading conditions. Their use in the practical field is unfortunately limited due to the intrinsic complexity of the mathematical formulation, the high number of parameters to be defined and the overall onerous computational demand. In the attempt to overcome some of the above difficulties, the paper explores the potentiality of an “advanced” constitutive soil model, which is not too complicated and demanding, to investigate the kinematic response of piles embedded in stratified deposits. This model, originally developed to describe the cyclic response of metals, in the geotechnical field has been already adopted by Anastasopoulos et al. (2011) to investigate the seismic response of shallow foundations. Referring to the above literature reference for details on the mathematical aspects of the constitutive law, only the key aspects are hereafter summarized.

The model is elasto-plastic with Von Mises failure criterion (Figure 1a), equipped with isotropic and kinematic hardening (Figure 1b). The evolution of the yield surface in the stress space is regulated by two components: a nonlinear kinematic hardening component, which describes the yield surface translation, and an isotropic hardening component, which governs the size of the

yield surface in dependence of the plastic strain,  $\bar{\varepsilon}^{pl}$ . In particular, the kinematic hardening component is defined as combination of a purely kinematic term, called Ziegler (1959) linear hardening, and a relaxation term, which introduces nonlinearity:

$$\dot{\alpha} = C \frac{1}{\sigma_0} (\sigma - \alpha) \dot{\varepsilon}^{pl} - \gamma_{kh} \alpha \dot{\varepsilon}^{pl} \quad (1)$$

In eq. (1)  $\alpha$  is the back-stress component,  $C$  represents the initial kinematic hardening modulus  $C = \sigma_y / \varepsilon_y = E = 2(1 + \nu)G_0$ ;  $\sigma_0$  is the equivalent stress defining the size of the yield surface and regulating the isotropic hardening;  $\gamma_{kh}$  is an internal parameter determining the rate of decrease of the kinematic hardening with accumulation of the plastic strain. Anastasopoulos et al. (2011) propose simple correlations to easily define the main parameters of the model, i.e. the maximum yield stress  $\sigma_y$  which defines the size of the limit surface and the parameter  $\gamma_{kh}$  that controls the kinematic hardening.

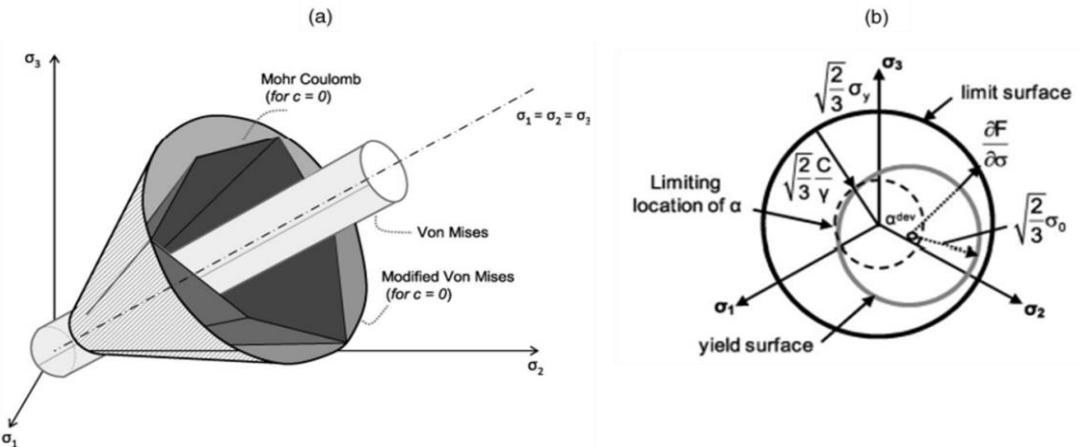


Figure 1. Adopted constitutive model: Von Mises yield surface in the principal stress space (a); projection on the  $\pi$ -plane (b) (Anastasopoulos et al., 2011)

In this paper the authors have calibrated the aforementioned parameters by simulating soil element response in cyclic simple shear test (at a confining pressure of 50 kPa) and comparing the numerical  $G/G_0-\gamma$  curves to the experimental ones published by Ishibashi and Zhang (1993). Elastic stiffness at small strains corresponds to a soil with Vs 100 m/s or 400 m/s. By fixing all the other parameters of the model (Table 1), the parameter regulating the kinematic hardening,  $\gamma_{kh}$ , and the elastic shear strain threshold,  $\gamma_{el}$ , were iteratively varied to reach the best fitting of the reference curve (Figure 2), obtained with the combination of parameters  $\gamma_{el} = 1 \cdot 10^{-4}$  and  $\gamma_{kh} = 5500$  (Figure 2).

Figure 3 shows the predicted hysteresis loops for different maximum-imposed shear strain ( $\gamma_{max}$  from  $1 \cdot 10^{-5}$  to  $1 \cdot 10^{-2}$ ).

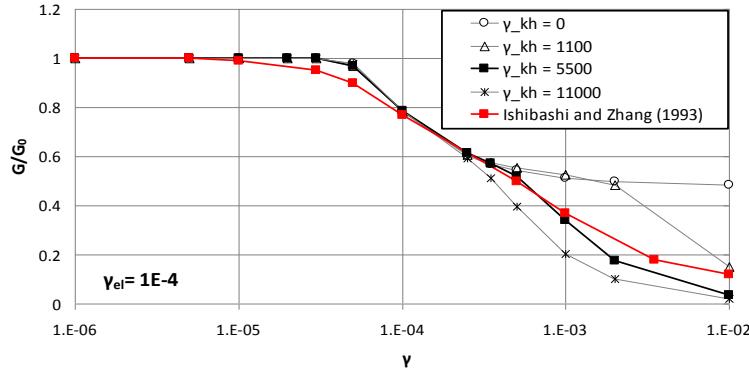


Figure 2. Model calibration in terms of  $G/G_0$ - $\gamma$  curves: numerical vs. experimental data of Ishibashi and Zhang (1993) for a confining pressure of 50 kPa

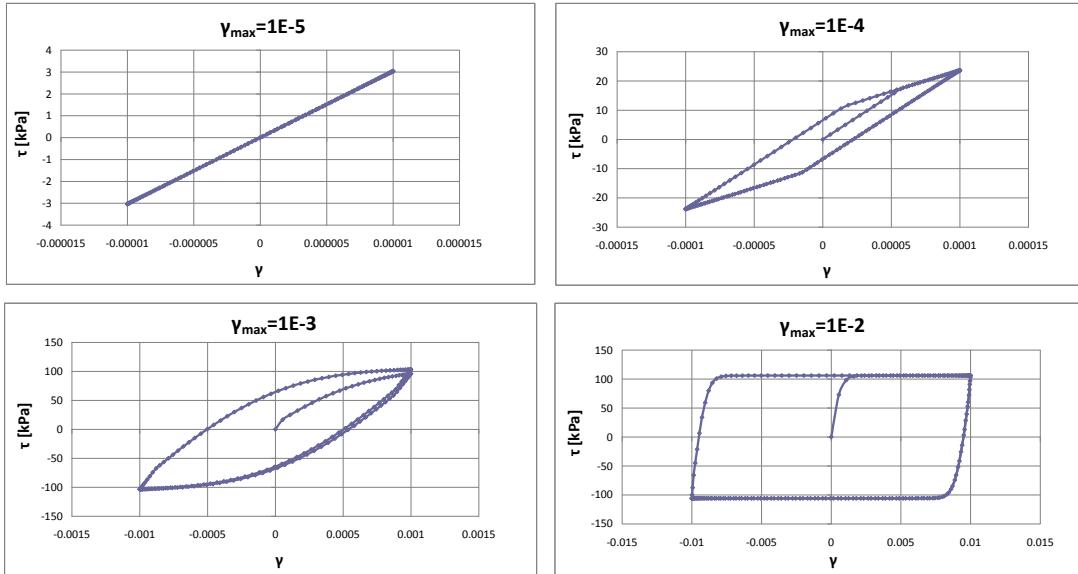


Figure 3. Hysteresis loops predicted at different strain level. Predictions refer to  $\gamma_{el} = 1 \cdot 10^{-4}$  and  $\gamma_{kh} = 5500$

### Boundary value problem

The studied problem is depicted in Figure 4. It consists of a cylindrical concrete pile of diameter 0.6 m and length of 20 m, embedded in a stratified soil deposit with two layers of different stiffness (the upper layer of thickness  $H_1=15$  m has a  $V_s=100$  m/s; the bottom layer of thickness  $H_2=15$  m is stiffer with a  $V_s=400$  m/s). The pile head is restrained against rotation. The problem has been analyzed in 3D with a finite element code. The soil has been modelled by 8-noded solid brick elements; the pile by 3D Timoshenko beam elements. Soil behaviour was described by the non linear constitutive law previously illustrated; pile behaviour was assumed linear elastic. The adopted parameters are reported in Table 1.

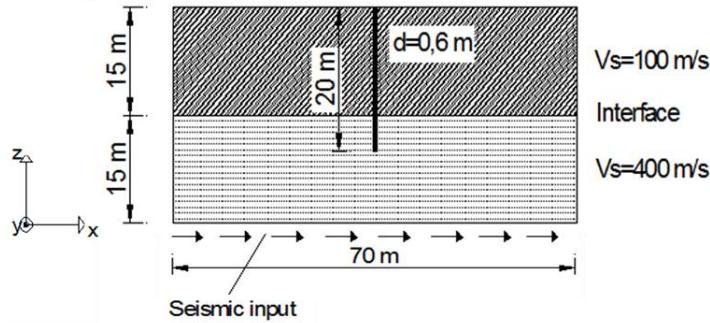


Figure 4. Overview of the model

Table 1. Soil parameters adopted in the analysis.

	Layer 1	Layer 2
Mass density $\rho$ [kg/m <sup>3</sup> ]	1900	1900
Poisson's ratio $\nu$ [-]	0.4	0.4
Young's modulus $E$ [kPa]	53200	851200
Initial shear stiffness modulus $G_0$ [kPa]	$19 \cdot 10^3$	$304 \cdot 10^3$
Shear wave velocity $V_s$ [m/s]	100	400
Initial kinematic hardening modulus $C$ [kPa]	$C=E=53000$	$C=E=850000$
Kinematic hardening parameter $\gamma_{kh}$	5500	5500
Yield stress at zero plastic strain $\sigma_0 = G_0 \gamma_{el}$ [kPa]	1.9	304
Elastic threshold $\gamma_{el}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$

The system was excited by natural accelerograms, applied in the x-direction (Figure 4). The analyses in the time domain were carried out by setting a time step increment  $\Delta t = 0.002$  s. During the dynamic stage the translational degrees of freedom along y and z have been fixed both at the base and on the lateral boundaries of the model.

In addition, soil and pile are connected by “tie” constraints, which are able to connect two distinct surfaces according to a master and slave formulation. In this case the nodes of the soil (slave) are forced to follow the displacements of the pile (master). This type of constraint may be updated if slippage or gap between pile and soil has to be accounted for. Coulomb friction model is used to describe the interaction between the contacting surfaces. These will not slip until the shear stress across their interface equals the limiting frictional shear stress.

The tie constraint has been also adopted during the seismic analysis to force the two vertical boundaries of the model in the zy plane (at x=0 and x=70 m) to undergo the same motion in x-direction.

## Numerical results

As starting point of the overall simulation process, a validation of the numerical procedure was carried out in the hypothesis of linear viscoelastic soil behaviour by comparing the results obtained by the developed 3D f.e.m. model with the results of a BDWF formulation,

implemented in the SPIAB code (Mylonakis et al., 1997). The geometric model is the same of Figure 4 with the parameters regulating the elastic behaviour listed in Table 1, with addition of a damping value of 10% for both layers. A Rayleigh damping formulation was adopted ( $[D] = \alpha[M]+\beta[K]$ ) with parameters  $\alpha = 1.5$  and  $\beta = 0.005$ . The adopted input motions are natural accelerograms recorded on rock, selected from the Italian database SISMA and scaled to a common peak value of 0.35g (Sica et al., 2011). For the scopes of this validation, the accelerograms were directly applied at the base of the soil column, without de-convolution or consideration for rock outcrop effects.

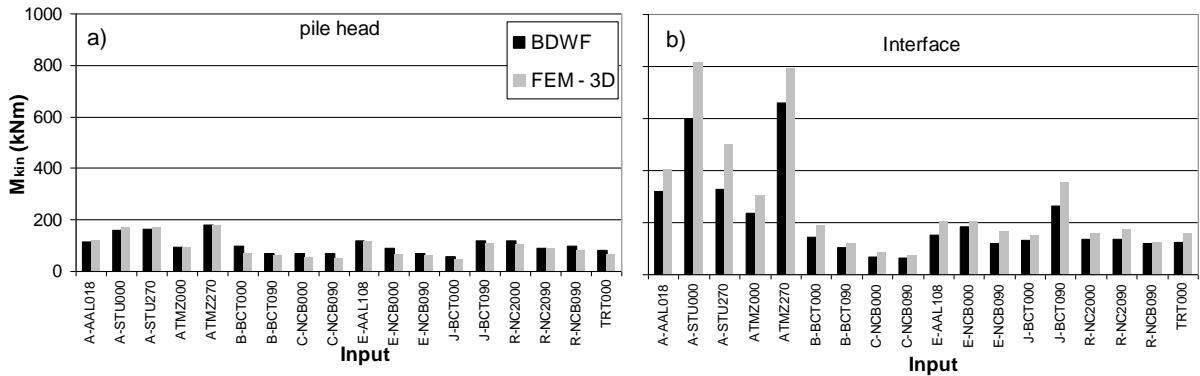


Figure 5. Validation of the 3D f.e.m. model in the hypothesis of linear visco-elastic behaviour of the soil: kinematic bending moments at the pile head (a) and at the layer interface (b) for different input motions all scaled at a peak acceleration of 0.35g.

After validating the 3D model in the elastic range (Figure 5), the kinematic hardening constitutive model described in the previous paragraph has been assigned to both soil layers of the considered deposit. All model parameters are reported in Table 1. When the advanced constitutive law was implemented, no further damping was added to the system (i.e., the Rayleigh damping previously adopted to compare the results shown in Figure 5 was removed). With reference to the signals A-TMZ270 and TRT000, Figure 6 shows the free-field peak acceleration and the maximum pile bending moment vs. depth  $z$ , provided by the linear elastic (LE) and the elasto-plastic (EP) analyses. It can be observed that for the considered signals, the kinematic bending moments provided by the 3D elasto-plastic analyses may be much more severe than those provided by the linear viscoelastic computation (Figure 5), not only at the layer interface (at 15 m) but also along the pile shaft, especially in the softer layer. Due to soil nonlinearity, higher damping is mobilized in the upper layer so that a deamplification of the free-field accelerations occurs in it (Figure 6a). At the same time, the stiffness degradation and plastic response occurring in the upper layer causes a huge increase in kinematic pile bending all around the layer interface. This response may be justified considering that soil nonlinearity, even inducing an overall higher damping in the soil deposit, may be very detrimental for kinematic pile bending since it enhances shear stiffness contrast not only at the interface between the two soil layers but also inside a single layer if markedly different strain levels are mobilized in it. This statement is in accordance with previous studies performed by BDWF approaches where soil nonlinearity was roughly accounted for by the equivalent linear procedure (Sica et al., 2013).

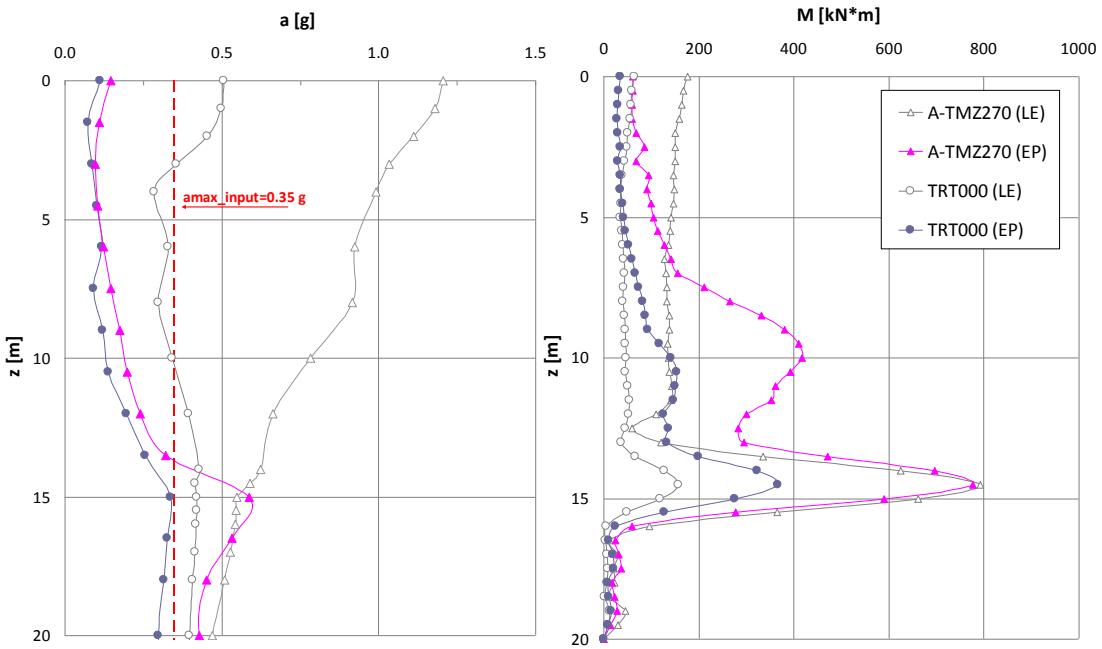


Figure 6. Maximum accelerations (a) and kinematic pile bending moments (b) vs. depth, computed assuming linear elastic (LE) or elasto-plastic (EP) soil behaviour, for the input signals labelled A-TMZ270 and TRT000, scaled to 0.35g.

To investigate the role of soil nonlinearity, the  $\tau$ - $\gamma$  loops provided by the 3D f.e.m analyses with the advanced Von Mises constitutive soil model and the A-TMZ270 accelerogram as input motion, have been plotted (Figure 7). They highlight the different response of the upper and lower soil layers. In particular, the upper layer denotes a marked plastic response while the bottom layer remains almost in the linear elastic range during all the shaking.

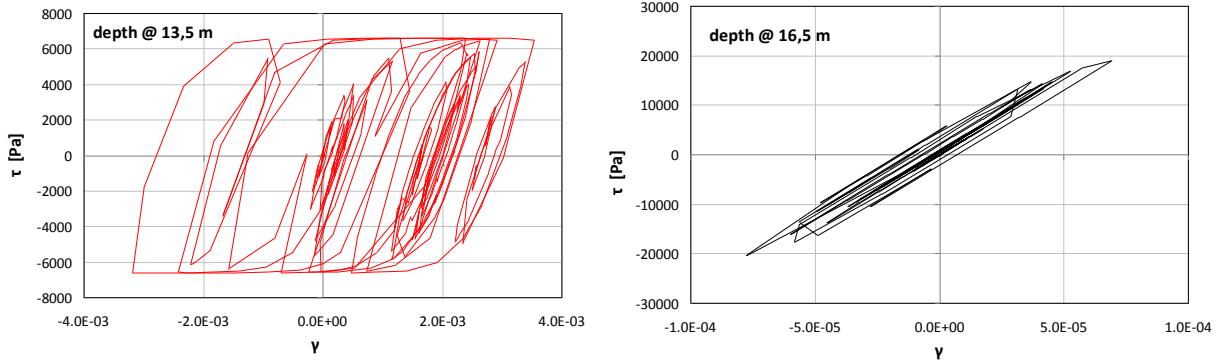


Figure 7. Hysteresis loop  $\tau$ - $\gamma$  provided by the 3D f.e.m analysis at two different depths, above and below the soil layer interface (placed at 15 m below the g.l.). The input signal is A-TMZ270 scaled to 0.35g.

## Conclusions

Kinematic bending of piles in layered soils was investigated through a 3D f.e.m. model

accounting for non linear, hysteretic and plastic behaviour of the soil under earthquake loading. A kinematic hardening constitutive soil model with Von Mises failure criterion was adopted. Even if the selected elasto-plastic model cannot be considered exhaustive and rigorous to represent the complex response of soil under cyclic loading, the paper highlights that soil nonlinearity may have detrimental effects on kinematic pile bending, depending on the shear strains (elastic and plastic) mobilized in the soil deposit by the earthquake.

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