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# Empirical Curve-Fitting Parameters for a Porewater Pressure Generation Model for Use in 1-D Effective Stress-Based Site Response Analysis

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## ABSTRACT

The Vucetic and Dobry porewater pressure (PWP) generation model has been widely implemented in site response analysis software such as DMOD and DEEPSOIL. This PWP model employs a relatively simple relationship between excess porewater pressure ratio, shear strain amplitude, and number of cycles, and requires only three curve-fitting parameters from undrained cyclic shear tests. In this study, empirical correlations for the three curve-fitting parameters are proposed, allowing the model to be used for site response analysis without complex laboratory testing. These correlations apply to subangular to subrounded clean sands, and use only relative density ($D_r$) and the uniformity coefficient ($C_U$). The correlations were calibrated using undrained cyclic DSS and triaxial tests performed on different sands at various $D_r$ values, and validated using tests not used for calibration. Computed PWP using the proposed correlations were assessed by mean residuals and coefficient of determination $r^2$, and yielded reasonable agreement between laboratory-measured and computed PWP.

## Introduction

Excess porewater pressure (PWP) generation during cyclic loading (e.g., earthquake shaking and pile driving) is of great interest to civil engineers as the increase in PWP reduces effective stress and thus soil strength, which is especially important for site response analysis and liquefaction evaluation. Commonly, site response analysis and liquefaction evaluation are performed in a frequency domain, total-stress framework. However, excess PWP generation and concurrent strain-softening of the soil reduces soil stiffness and can significantly modify ground motion propagation through the soil (e.g., Youd and Carter 2003).

Recently, computational advances have reduced greatly the time required for performing site response analyses in a time domain, effective-stress framework, thereby allowing nonlinear, time-domain, effective stress site response analysis to be more widely used in practice. In this framework, a PWP generation model is coupled with a stress-strain constitutive model. In the past 40 years, several PWP generation models have been proposed, including cyclic stress-based models (e.g., Seed et al. 1975), cyclic strain-based models (e.g., Vucetic and Dobry 1986), and

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energy-based models (e.g., Green et al. 2000). This paper focuses on the Vucetic and Dobry (1986) strain-based PWP generation model as it has been implemented in widely-used codes for 1D site response analysis such as DMOD (Matasovic 1993) and DEEPSOIL (Hashash et. al 2010). However, the Vucetic and Dobry (1986) model curve-fitting parameters currently must be defined using a complex suite of cyclic strain-controlled laboratory tests or must be crudely estimated via judgment by comparison to curve-fitting parameters defined for a limited number of sands. No empirical correlations are available to estimate these parameters. The purpose of this paper is to: (1) develop correlations for these curve-fitting parameters using readily-defined soil properties; and (2) illustrate how cyclic stress-based tests can be used to derive the strain-based PWP generation model parameters.

## Vucetic and Dobry Model (1986)

Vucetic and Dobry (1986) developed an empirical relationship among excess PWP ratio ($r_u$ = $\Delta u/\sigma'_{vo}$, where $\Delta u$ = excess PWP and $\sigma'_{vo}$ = initial effective vertical stress), cyclic shear strain amplitude ($\gamma_c$), and number of loading cycles ($N_c$) based on the results of undrained, strain-controlled cyclic shear tests. This relationship is defined as follows:

$$r_{u,N} = \frac{pfN_cF(\gamma_c - \gamma_{tvp})^s}{1 + fN_cF(\gamma_c - \gamma_{tvp})^s} \quad (1)$$

where $r_{u,N}$ = residual excess PWP ratio at cycle $N_c$; f = 1 or 2 for one- or two-dimensional loading, respectively; p, F, and s = curve-fitting constants; and $\gamma_{tvp}$= volumetric threshold shear strain, which is defined as the shear strain below which no significant excess PWP is generated during cyclic loading, regardless of density or cyclic strain amplitude. This shear strain is usually between 0.01 and 0.02% for most sands (Dobry et al. 1982). Rearranging Equation 1 yields:

$$\frac{1}{r_{u,N}} = \frac{1}{p} + \frac{1}{pfN_cF(\gamma_c - \gamma_{tvp})^s} \quad (2)$$

Figure 1 illustrates the Vucetic and Dobry (1986) method to derive the parameters F, s and p. Figure 1(a) presents cyclic shear test data in terms of $1/N_c$ and $1/r_{u,N}$ to define 1/p and $g(\gamma_c)$, where $g(\gamma_c) = F(\gamma_c - \gamma_{tvp})^s$. Generally, 1/p can be determined uniquely for each sand. As $N_c$ increases, the second term on the right-hand side of Eq. (2) approaches zero, and thus the value of p defines the $r_u$ value measured at the end of a cyclic shear test. In this study, it is assumed that $r_u$ can eventually reach unity, even for dense soils, if $\gamma_c > \gamma_{tvp}$. Dobry (1985), Vucetic and Dobry (1986), and Matasovic (1993) have shown that $p \rightarrow 1$ in all cyclic shear tests. As such, $p \equiv 1$ for this study. For unidirectional shaking, f = 1. Therefore, for a constant $\gamma_c$, the inverse of the slope of the relation between $1/N_c$ and $1/r_{u,N}$ is $g(\gamma_c)$. Values of $g(\gamma_c)$ are then plotted against $\gamma_c$ (Figure 1b) to define F and s. When s = 1 (and $\gamma_{tvp}$ = 0.02%), the relation between $g(\gamma_c)$ and $\gamma_c$ becomes linear, and for values of $s \neq 1$, the relation between $g(\gamma_c)$ and $\gamma_c$ is nonlinear (Figure 1b). Based on the findings of this study and consistent with Matasovic (1993), selecting s = 1 provides a reasonable fit to the $g(\gamma_c)$ - $\gamma_c$ data. Defining p = 1 and s = 1 means that only the parameter F must be calibrated to soil properties. This calibration is described below.

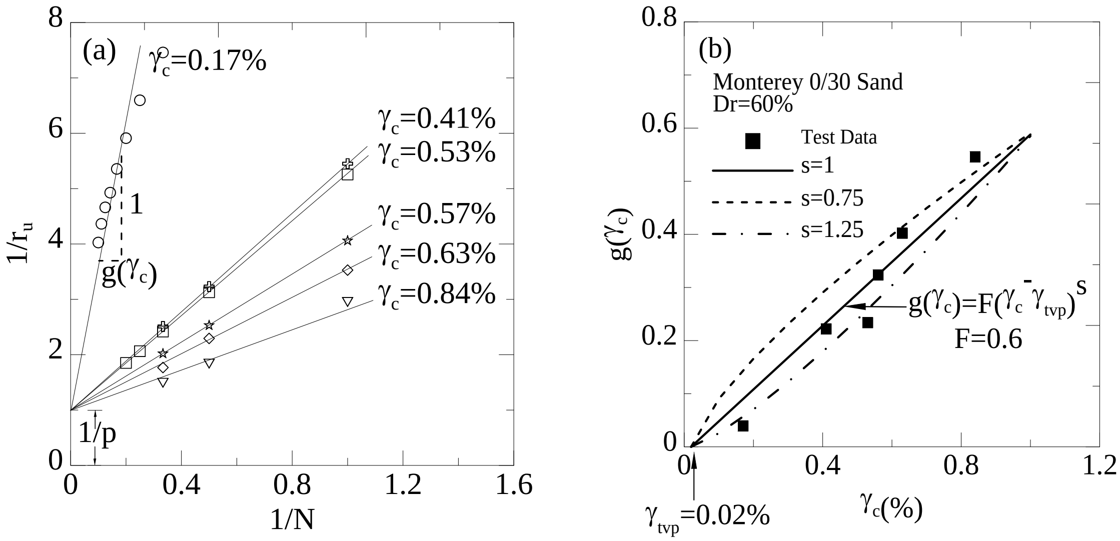


Figure 1: Determination of Vucetic and Dobry PWP generation model curve-fitting parameters for Monterey 0/30 sand at $D_r$=60% (data from Wu et al. 2003 and Polito 1999). (a) derivation of $g(\gamma_c)$ and curve-fitting parameter p; and (b) derivation of curve-fitting parameter s and F.

## Calibration Database, Development, and Validation

A total of 123 cyclic shear test results were compiled from the literature for developing a correlation for the curve-fitting parameter, F. These include cyclic direct simple shear (cDSS) and cyclic triaxial compression (cTC) tests (Table 1). Several index properties were considered in developing a correlation for F. Two index properties, $D_r$ and coefficient of uniformity, $C_U$, were strongly correlated to PWP generation. Relative density has been associated with PWP generation by many researchers (e.g., Dobry 1982; Kenan 2005) and is commonly used in many PWP generation models (e.g., Polito et al. 2008; Green et al. 2000; Cetin et al. 2012). Generally, for tests under the same conditions (i.e., consolidation stress, $\gamma_c$, $N_c$), as $D_r$ increases, a lower $r_u$ and rate of PWP generation are expected, as illustrated in Figure 2(a).

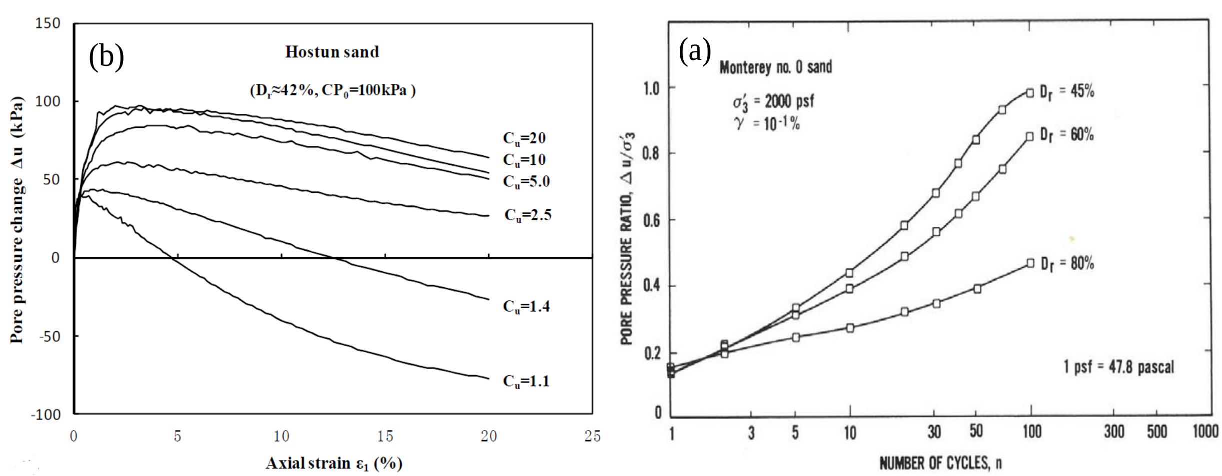


Figure 2: (a) Effect of $D_r$ on porewater pressure generation (Dobry1982); and (b) influence of $C_U$ on porewater pressure generation (Li 2013).

The coefficient of uniformity ($C_U = D_{60}/D_{10}$, where $D_{60}$ and $D_{10}$ are the grain diameters corresponding to 60% and 10% finer by weight, respectively) also affects cyclic soil behavior. For example, Monterey and Yatesville sands have similar particle shapes, yet in otherwise identical tests, the Yatesville sand with a higher value of $C_U$ generates higher $r_u$ values at a given number of cycles. Similarly, Li (2013) performed numerous undrained TC tests on Hostun sand and glass balls to investigate the influence of changes in gradation on soil behavior. Saturated Hostun sand and glass ball specimens were prepared by moist tamping at identical $D_r$ and consolidation stresses, but were reconstituted from samples with different values of $C_U$ ranging from 1.1 to 20. As is shown in Figure 2(b), for Hostun sand specimens with $D_r$ = 42%, changes in Cu greatly influence PWP generation, with higher values of $C_U$ generating higher values of $r_u$. Undrained TC tests performed by Castro (1982) on Banding sand specimens with different values of $C_U$ yielded similar results. While this result may seem counter-intuitive, it can be understood by considering that well-graded sands (high $C_U$) will sediment at a higher $D_r$ than poorly-graded sands (low $C_U$) at a given depositional energy. Therefore, when sand gradations of various $C_U$ values are prepared to the same value of $D_r$, the well-graded sand will be more contractive than the poorly graded sand.

Table 1: Summary of database for development and validation of proposed correlation.

| Sand | Angularity | $C_U$ | No. & type of tests | $D_r$ (%) | $\sigma'_c$ (kPa) | Reference |
|---|---|---|---|---|---|---|
| Monterey 0/30 | Subangular - subrounded | 1.4 | 62 σ-controlled cDSS | 30 - 80 | 40, 80, 180 | Wu et al. (2003) |
| Monterey 0/30 | Subangular - subrounded | 1.4 | 16 σ-controlled cTC | 30 - 80 | 100 | Polito (1999) |
| Yatesville | Subangular - subrounded | 2.4 | 25 σ-controlled cTC | 30 - 70 | 100 | Polito (1999) |
| Wildlife sand B | n/a | 2.8 | 13 ε-controlled cDSS & cTC | 25 | 96 | Vucetic & Dobry (1996) |
| Heber Rd. PB sand | n/a | 2.3 | ε-controlled cDSS & cTC | 45 | n/a | Vucetic & Dobry (1996) |
| SMB sand | n/a | 1.8 | 7 ε-controlled cDSS | 75 | 200 | Matasovic (1993) |
| Monterey 0/30 | Subangular - subrounded | 1.4 | 9 ε-controlled cDSS | 32, 50, 93 | 100 | Kenan (2005) |
| Sacramento River | Subangular - subrounded | 1.4 | 4 σ-controlled cDSS | 35, 45, 55 | 207 | Boulanger et al. (1991) |
| Fraser River | Subangular - subrounded | 1.6 | 2 σ-controlled cDSS | 40, 80 | 100 | Sriskandakumar (2003) |
| Ticino | Angular | 1.5 | 1 σ-controlled cDSS | 75 | 100 | Porcino (2007) |

Effective vertical stress was not included in the correlation because: (1) most of the tests in the laboratory database were performed at a consolidation stress of approximately 100 kPa; and (2) as shown in Figure 3, in cyclic direct simple shear tests performed on Monterey 0/30 sands under $\sigma'_{vc}$ of 40, 80, and 180 kPa, Wu et al. (2003) illustrated that the change in liquefaction resistance with increasing consolidation stress was minor for loose to medium dense sands ($D_r$ < 60%). At $D_r$ = 80%, liquefaction resistance increases by approximately 30% at low $\sigma'_{vc}$. Sufficient data to examine this issue in more detail were not available in the literature; therefore, effective vertical stress was not included in the correlation. As a result, the proposed correlation will be conservative (overestimate PWP generation) in very dense sands at low $\sigma'_{vo}$. Given this limitation, this issue warrants further research.

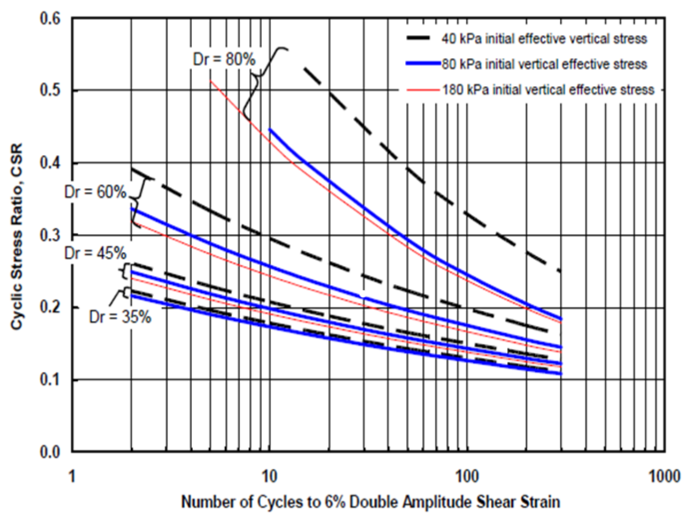


Figure 3: Influence of effective vertical stress on liquefaction resistance (Wu et al. 2003).

Based on the rationale above, a correlation among F, $D_r$, and $C_U$ was developed. The correlation was guided by the observation that the value of F, which reflects the rate of PWP generation, should be inversely related $D_r$ and directly related to $C_U$. Tests on Monterey and Yatesville sands were individually analyzed to compute values of F, while values of F reported by Matasovic (1993) for SMB sand and by Vucetic and Dobry (1986) for Heber Road and Wildlife Site sands were used directly. Figure 4(a) presents the proposed correlation. Note that each data point in Figure 4(a) represents a number of specimens tests at the same $D_r$ and consolidation stress and were interpreted using the method shown in Figure 1. The correlation applies to clean, subangular to subrounded silica sands, as sufficient data for sands with fines, other grain shapes, and other mineralogies were not available in the literature.

As described earlier, the Vucetic and Dobry (1986) model was developed using strain-controlled tests; however, most of the database consists of stress-controlled tests. In many stress-controlled tests, the first several stress cycles yield similar shear strains (i.e., the difference between the minimum and maximum $\gamma_c$ < 30%), allowing the initial stress cycles to be treated as a strain-controlled test. Matasovic (1993) used a similar approach in deriving Vucetic and Dobry (1986) model parameters from tests that, as described by Matasovic (1993), were not "perfectly strain-controlled." Using the initial cycles, Matasovic (1993) defined F for SMB sand. To test this approach, the proposed correlation (developed using stress-controlled tests) was compared to strain-controlled test data from Kenan (2005; Table 1). Figure 4(b) presents the $\gamma_c$-g($\gamma_c$) test data with the F-values predicted by the proposed correlation. The good agreement between the correlation and data confirms that stress-controlled can be used to derive the correlation for F.

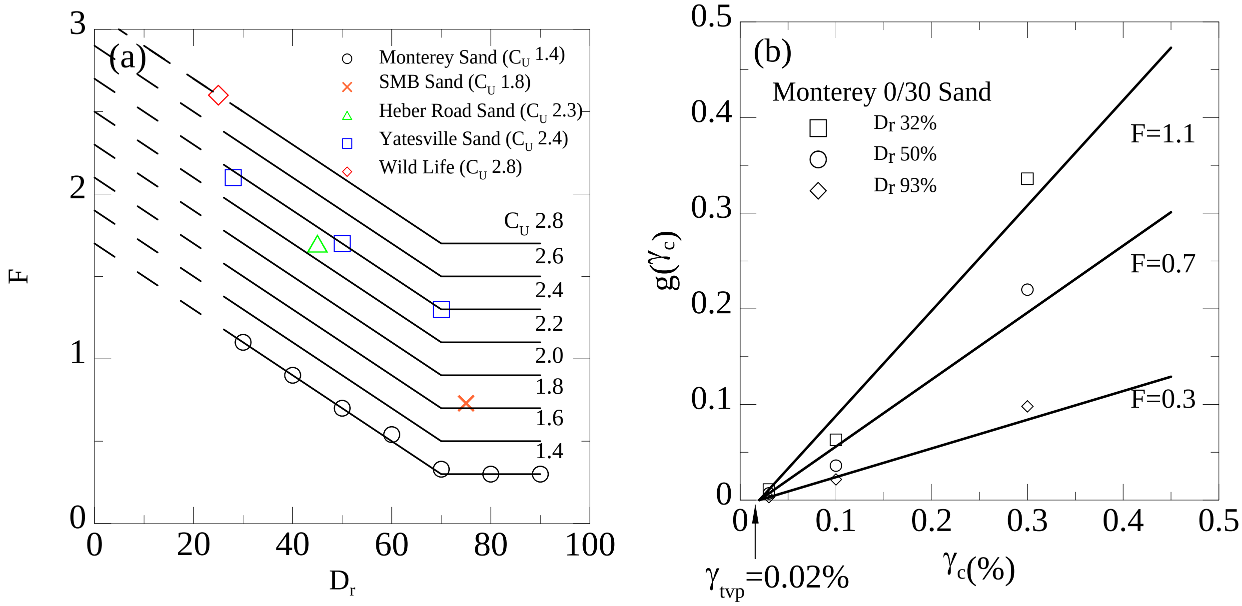


Figure 4: (a) Proposed correlation to estimate parameter F; and (b) comparison of correlation derived from stress-controlled tests with data from strain-controlled tests from Kenan (2005).

The correlation was assessed using laboratory data from Boulanger et al. (1991), Sriskandakumar (2003), and Porcino (2007) (see Table 1), which were not used to develop the correlation. Values of F for these sands were defined using the proposed correlation (Figure 4a) and PWP was calculated using Eq. (2). Figure 5(a) presents an example comparison between measured and predicted PWP for Sacramento River sand. As is shown in the figure, the model reasonably predicts PWP generation for the Sacramento River sand.

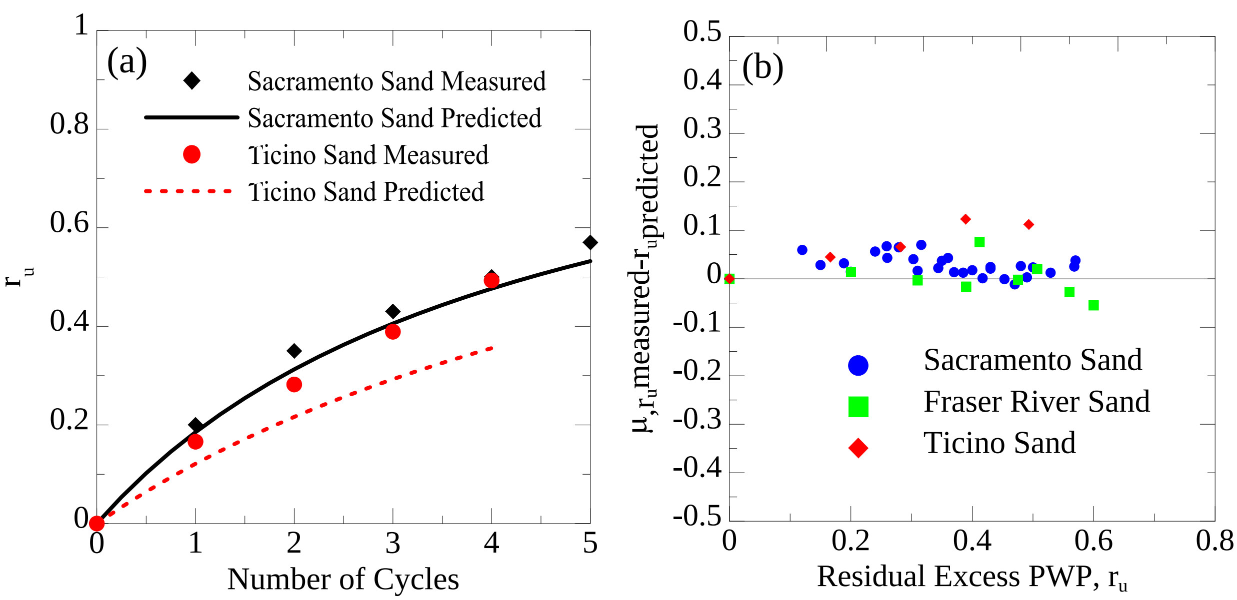


Figure 5: (a) Comparison of measured and predicted PWP for Sacramento River and Ticino sands; and (b) residual computed for Sacramento River, Fraser River, and Ticino sands.

Residuals, μ (= $r_{u,measured}$ - $r_{u,predicted}$) and coefficient of determination $r^2$ (Draper et al. 1966) were computed for $r_u$ for each test. (A residual of zero implies accurate and unbiased prediction and a higher $r^2$ implies a better goodness of fit.) The coefficient of determination was calculated using a single value of $r_u$ for each cycle for up to the first five cycles. The predicted and measured pore pressures are compared and $r^2$ were calculated. As is shown in Figure 5(b), residuals for PWP generally were less than 0.1, but may indicate a slight bias at low $r_u$ values, and residuals decreased as $r_u \rightarrow 1$. Values of $r^2$ in Table 2 also show reasonably good performance of the model. We note that the proposed correlation is not as successful for Ticino sand compared to the Sacramento and Fraser River sands. It is possible that the 5% mica content of Ticino sand (Baldi et al. 1982), which makes the sand relatively compressible, also affects the correlation for F. However, there were insufficient data available in the literature to evaluate the role of compressibility and mineralogy on PWP generation. Further research on these topics is warranted.

Table 2: Validation Data for the Correlation.

| Reference | Sand Type | $\mu_{min}$ | $\mu_{max}$ | $\mu_{mean}$ | $r^2$ |
|---|---|---|---|---|---|
| Boulanger et al. (1991) | Sacramento | 0.023 | 0.053 | 0.038 | 0.96 |
| Boulanger et al. (1991) | Sacramento | 0.011 | 0.089 | 0.054 | 0.86 |
| Boulanger et al. (1991) | Sacramento | 0.013 | 0.028 | 0.021 | 0.95 |
| Boulanger et al. (1991) | Sacramento | 0.001 | 0.08 | 0.02 | 0.93 |
| Sriskandakumar (2003) | Fraser River | 0.002 | 0.014 | 0.005 | 0.98 |
| Sriskandakumar (2003) | Fraser River | 0.02 | 0.09 | 0.04 | 0.94 |
| Porcino and Caridi (2007) | Ticino | 0.045 | 0.137 | 0.063 | 0.77 |

## Conclusions

In this study, the Vucetic and Dobry (1986) PWP generation model is briefly described and a correlation is developed to estimate model calibration parameters F, s, and p to allow the model to be more easily implemented in site response analysis software. The proposed correlation is based on sand index properties $D_r$ and $C_U$. The correlation was developed using stress-based tests, and the use of these tests to derive strain-based model parameters also is introduced and validated. Cyclic tests performed on different sands are used to validate the proposed correlation. These data illustrate that the proposed correlation yields reasonable estimates of PWP generation during cyclic shear for clean, subangular to subrounded silica sands. Fines content, grain shape, and mineralogy may affect the proposed correlation; however, there are insufficient data in the literature to address the role of these factors.

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