

# A Probabilistic Approach to Site-Specific, Hazard-Consistent Vertical Design Spectra

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## ABSTRACT

A probabilistic, site-specific, and hazard-consistent vertical-to-horizontal response spectral ratios (V/H ratios) model is proposed. Prominent features of the proposed model are 1) The vertical hazard is obtained as the convolution between horizontal hazard and V/H ratios; and 2) the correlation between V/H ratios and horizontal motions is considered in the convolution. Both epistemic and aleatory uncertainties in the V/H ratios are incorporated in a natural manner as in a conventional Probabilistic Seismic Hazard Analysis (PSHA). A case study is conducted using both deterministic and probabilistic approaches for a site located in the Central and Eastern United States (CEUS). The outcome shows that the proposed probabilistic model may result in lower vertical design spectra compared to a deterministic envelop approach. Limitations and potential applications of the new models in the practice are discussed in the context and concluding remarks.

## Introduction

Design Spectra and associated time histories in the vertical direction are required in the seismic design of certain critical structures such as nuclear reactor buildings (U.S. NRC, 2014b). Other candidates for which vertical shaking may be an important consideration in the seismic design include: buildings with long-span floors, floor slabs supporting sensitive equipment, industrial facilities supporting massive equipment, long cantilevers, and near-fault ordinary bridges (NIST, 2012; Gulerce and Abrahamson, 2011). A commonly used approach in current practice for developing vertical design spectra is scaling the horizontal design spectra using deterministic vertical-to-horizontal spectral acceleration (V/H) ratios. Following this approach, V/H ratios are either taken from a relevant design code/standard, determined from the mean/median prediction of an empirical model, or taken as the envelop of predictions from multiple empirical models. Alternatively, vertical design spectra may be determined directly using site-specific vertical hazard curves, which can be derived using a site-specific hazard-consistent probabilistic approach. Based on a brief review of commonly used V/H models, one implementation of the probabilistic approach is presented in this study. The proposed model has formal representations of different uncertainties, similar to those in a conventional probabilistic seismic hazard analysis (PSHA). A case study is conducted for a site located in Central and Eastern United States (CEUS). The vertical design spectra developed using alternative approaches are compared.

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## Review of the Currently Used Approaches to Vertical Design Spectra

Broadly speaking, there are two approaches to developing vertical design spectra (Gülerce and Abrahamson, 2011): (1) compute the vertical hazard directly using ground motion prediction equations (GMPEs) for the vertical component of ground motion following the same PSHA procedures used for the horizontal component, or 2) scale the horizontal design spectra using a vertical-to-horizontal response spectral ratio (V/H ratio) model. The first approach, although theoretically sound, was previously subject to the scarcity of vertical GMPEs relative to the amount of horizontal GMPEs. The situation seems to be improved with publications of more vertical GMPEs (ex. PEER, 2013). In addition, conducting horizontal and vertical PSHA separately may result in different controlling scenarios for horizontal and vertical components. The second approach bypasses the inconsistency in the controlling scenarios by assuming the same controlling scenarios for both horizontal and vertical hazard. The second approach also implicitly presumes equal aleatory uncertainty in the horizontal and vertical hazard. In most applications, the V/H ratios are applied to the Uniform Hazard Spectra (UHS). Gülerce and Abrahamson (2011) illustrated an application applying V/H ratios to Conditional Mean Spectra (CMS).

Bommer et al. (2011) grouped the existing V/H models into three categories: (1) Code/Standard or regulatory guide based; (2) independent predictions of spectral ordinates of horizontal and vertical components, which allow the median of V/H ratio to be calculated for certain scenarios; (3) direct predictions of V/H ratios, which are usually accompanied with associated uncertainties.

The first V/H ratio model, i.e. V/H ratio from a Code/Standard or regulatory guide, is commonly used in lieu of site-specific seismic hazard studies. Representative V/H ratios from various sources are illustrated in Figure 1. Some give only a single V/H value [NEHRP (2009) and RG 1.60 (U.S. NRC, 2014a)], while others recommend scenario dependent V/H ratios (Eurocode 8 and ASCE 4-98). NUREG/CR-6728 provides V/H ratios for a range of peak ground acceleration (PGA). As noted by Bommer et al. (2011), a key shortcoming of these simplified V/H ratio models is their inability to capture the strong influence of source-to-site distance.

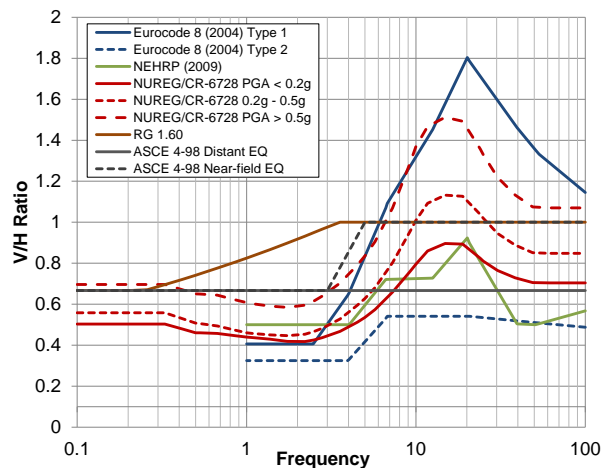


Figure 1. Representative V/H ratios from code/standard/regulations for Western North America conditions

V/H models based on either independent predictions of spectral ordinates or direct predictions of V/H ratios are preferred for site-specific seismic hazard studies. A summary of these models is presented in Bommer et al. (2011). One approach to apply these models is to take an envelope of the V/H ratios calculated using alternative empirical V/H models for different controlling scenarios for the annual frequency of exceedance (AFE) of interest. A limitation of this approach is the lack of formal representation of uncertainties; as such, this type of approach is denoted as the deterministic envelope approach in this study. It should be noted that the deterministic approach, although has limitations, requires minimum efforts of post-processing on the outcomes of a horizontal PSHA; and therefore is sometimes considered in the industry applications as a relatively conservative representation for V/H model. The major motivation to develop the new probabilistic V/H model is to incorporate both epistemic and aleatory uncertainties in a similar manner as in a conventional PSHA for the horizontal hazard. The correlation between V/H ratio and horizontal motion is also considered in the proposed model.

### **Probabilistic Approach to Site-Specific, Hazard-Consistent Vertical Design Spectra**

The proposed model is analogous to the convolution method (Bazzurro, 1998), for incorporating site amplification in PSHA (McGuire et al, 2001). The essence of convolution approach is that the spectral accelerations (SAs) at elevation of interest, which are needed in the hazard integration, are evaluated as the convolution of SAs at reference rock and site amplification factors. In the proposed model, the vertical hazard is integrated utilizing the convolution between horizontal SAs and site-specific V/H ratios. The correlation between V/H ratio and horizontal motion is also considered in the formulation. In this sense, the proposed model is considered as an application in the framework of Vector-Valued PSHA (Bazzurro, 1998). The formulation of the proposed model is presented in the following paragraphs.

The annual frequency of exceedance ( $\lambda(a^V)$ ) for a target vertical spectral acceleration level ( $a^V$ ) at a control point, denoted as vertical soil hazard hereby, can be expressed as the summation of deaggregated hazard from different magnitude ( $M_i$ ) and distance ( $R_j$ ) bins

$$\lambda(a^V) = \sum_i \sum_j \lambda(a^V | M_i, R_j) \quad (1)$$

In each magnitude and distance bin, the vertical soil hazard is expressed as the convolution between horizontal soil hazard and V/H ratio.

$$\lambda(a^V | M_i, R_j) = \sum_k P(R > \frac{a^V}{a_k^H} | M_i, R_j, a_k^H) \lambda'(M_i, R_j, a_k^H) \Delta a_k^H \quad (2)$$

in which,  $R$  is V/H ratio;  $P(R > \frac{a^V}{a_k^H} | M_i, R_j, a_k^H)$  is the probability of  $R$  greater than  $\frac{a^V}{a_k^H}$  in the  $M_i, R_j$  bin given horizontal spectral acceleration  $a_k^H$ ;  $\lambda'(M_i, R_j, a_k^H) \Delta a_k^H$  is the annual probability of occurrence of acceleration  $a_k^H$  at the control point, which is obtained by differentiating the deaggregated horizontal soil hazard curve in the  $M_i, R_j$  bin; subscript “ $k$ ” denotes  $k^{\text{th}}$  differentiation point on deaggregated hazard curve.

The correlation between logarithmic V/H ratio ( $\ln(R)$ ) and logarithmic horizontal acceleration

( $\ln(H)$ ) is considered in evaluating  $P(R > \frac{a^V}{a_k^H} | a_k^H)$  ( $M_i, R_j$  are omitted for brevity). By assuming a joint normal distribution for  $\ln(R)$  and  $\ln(H)$ ,  $P(R > \frac{a^V}{a_k^H} | a_k^H)$  is calculated as  $1 - \Phi(\epsilon_{R|H})$  with  $\Phi(\cdot)$  being the normal cumulative distribution function.  $\epsilon_{R|H}$  is expressed as

$$\epsilon_{R|H} = \frac{\ln\left(\frac{a^V}{a_k^H}\right) - (\mu_{\ln R} + \rho_{\ln R, \ln H} \sigma_{\ln R} \epsilon_H)}{\sigma_{\ln R} \sqrt{1 - \rho_{\ln R, \ln H}^2}} \quad (3)$$

in which,  $\rho_{\ln R, \ln H}$  is the correlation coefficient between  $\epsilon_{R|H}$  and  $\epsilon_H$ .  $\epsilon_H$  is evaluated as  $\epsilon_H = (\ln(a_H) - \mu_{\ln H}) / \sigma_{\ln H}$ .  $\mu_{\ln H}$  is equal to  $\mu_{\ln H_0} + \mu_{\ln AF}$  with  $H_0$  being the horizontal rock acceleration and  $\sigma_{\ln H}$  is approximated as  $\sqrt{(c_1 + 1)^2 \sigma_{\ln H_0}^2 + \sigma_{\ln AF}^2}$  (Bazzurro and Cornell, 2004) with  $c_1$  being the slope of  $\ln(AF)$  (logarithmic horizontal site amplification factors) versus  $\ln(H_0)$  relation.  $\mu_{\ln H_0}$  and  $\sigma_{\ln H_0}$  are determined using horizontal GMPEs for certain controlling scenarios ( $M_i, R_j$ ). The weights from GMPEs logic tree should be considered for  $\mu_{\ln H_0}$  and  $\sigma_{\ln H_0}$  determination.

In summary, the following steps are involved in developing the vertical soil hazard:

- 1) Develop horizontal reference rock hazard and associated deaggregation using appropriate Seismic Source Characterization (SSC) model and Ground Motion Characterization (GMC) model;
- 2) Compute site-specific horizontal site amplification factors for a range of AFE for controlling  $M_i, R_j$  scenarios;
- 3) Develop horizontal soil hazard as convolution between horizontal rock hazard and site amplification factors. A detailed guidance for steps 2) and 3) is presented in EPRI (2013b);
- 4) Estimate the mean and standard deviation for horizontal motion ( $\mu_{\ln H}$  and  $\sigma_{\ln H}$ ) for controlling scenarios for each AFE of interest;
- 5) Estimate mean and standard deviation for V/H ratios ( $\mu_{\ln R}$  and  $\sigma_{\ln R}$ ) for controlling scenarios for each AFE of interest using appropriate empirical V/H models and site parameters;
- 6) Compute epsilon of V/H ratios ( $\epsilon_{R|H}$ ) conditional on the epsilon of horizontal ground motion (Equation 3);
- 7) Apply the convolution between V/H ratio and horizontal soil hazard to get vertical soil hazard for controlling  $M_i, R_j$  scenarios (Equation 2);
- 8) Aggregate the vertical soil hazard (Equation 1).

These steps are applied frequency by frequency. The desired vertical design spectra for the control point are then interpolated from vertical soil hazard for the AFE of interest.

It should be noted that the same formulations for the convolution between horizontal soil hazard and V/H ratios are also applicable for the convolution between horizontal rock hazard and site

AFs. However, the correlation between H and R can be (conservatively) neglected in practice because they are mildly negatively correlated (Bazzurro and Cornell, 2004).

As mentioned earlier, the controlling scenarios for each of the intensity measures (IMs) ( $H_0$ , H, V, etc.) may be different. The difference in the controlling scenarios certainly has implications in the hazard analysis and succeeding design spectra development. In this sense, the proposed approach is still an approximation by not incorporating those implications. Nevertheless, it is useful for refining the vertical design spectra when a formal vertical PSHA is not available, which is often the case in practice.

### **Case Study for a CEUS Site**

A case study is presented in this section for a nuclear power plant (NPP) site located in Central and Eastern United States (CEUS). For this site, the horizontal rock hazard is developed using the SSC model in EPRI, U.S. DOE and U.S. NRC (2012) and GMC model in EPRI (2013a). Site-specific horizontal site amplification factors and horizontal soil hazard are developed using convolution approach following the guidance in EPRI (2013b). Two alternative soil profiles (denoted as A and B) are considered as representation for epistemic uncertainty. Both deterministic and probabilistic approaches are used in developing V/H ratios and vertical design spectra.

Five different V/H models are considered in the deterministic approach: 1) V/H ratios for Central and Eastern United States (CEUS) from McGuire et al. (2001); 2) V/H ratios from for Western United States (WUS) from McGuire et al. (2001); 3) V/H models from Abrahamson and Silva (1991) [AS97]; 4) V/H models from Bozorgnia and Campbell (2004) [BC04]; and 5) V/H models from Gülerce and Abrahamson (2011) [GA11]. The controlling events are derived following the guidance in U.S. NRC (2007) for both low frequency (LF) and high frequency (HF) for AFE of  $1E-4$  because the design spectra are governed by  $1E-4$  UHS. The empirical V/H models developed in AS97, BC04 and GA11 are developed based on a ground motion database dominated by the records in WUS. A transfer function, as suggested in McGuire et al. (2001) [denoted as “shifted” in Figure 2] is applied to these V/H ratios before use for the CEUS site. For the deterministic approach, the recommended V/H ratio is taken as the envelop of all the V/H ratios under consideration.

All the V/H models used in the deterministic approach, except RG 1.60 are also adopted in the probabilistic approach. Alternative models for median are represented in a logic tree as shown in Figure 3. The model for standard deviation and correlation are taken from GA11.

Frequency dependent median V/H and standard deviation of  $\ln(V/H)$  are developed for controlling scenarios for 11 AFEs (Figure 4). The frequency dependent correlation coefficients between V/H ratios and horizontal motions are developed following the suggestions in Gülerce and Abrahamson (2011) for all the 11 AFEs. The correlation coefficient for a representative AFE ( $9.19E-5$ ) is presented in Figure 5. Different from the application in scaling CMS, only the correlation coefficients for the same frequencies are used in this study.

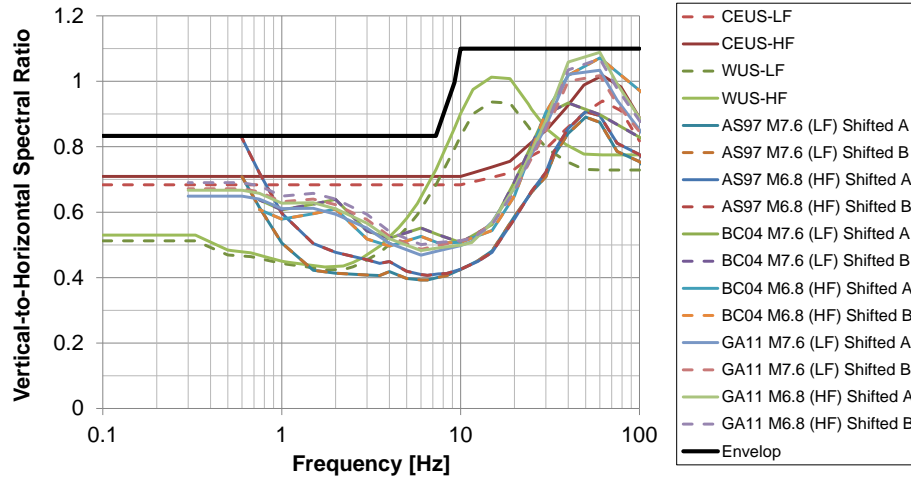


Figure 2. V/H ratios developed from deterministic envelop approach (the black solid line denoted as Envelop is the recommend V/H ratio from deterministic approach)

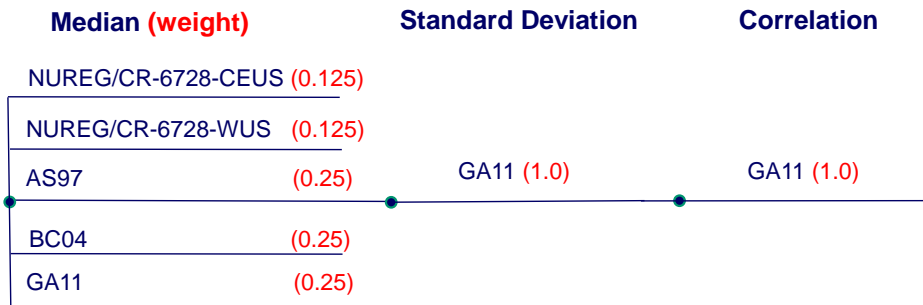


Figure 3. Logic tree for the probabilistic approach

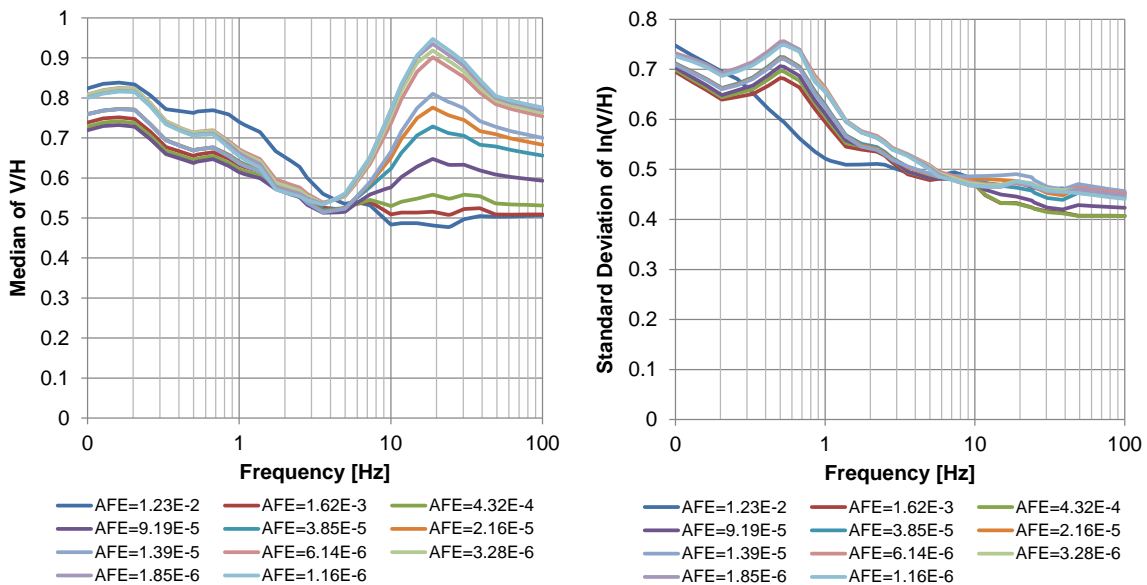


Figure 4. Median of V/H ratio and standard deviation of logarithmic V/H ratio for 11 AFEs

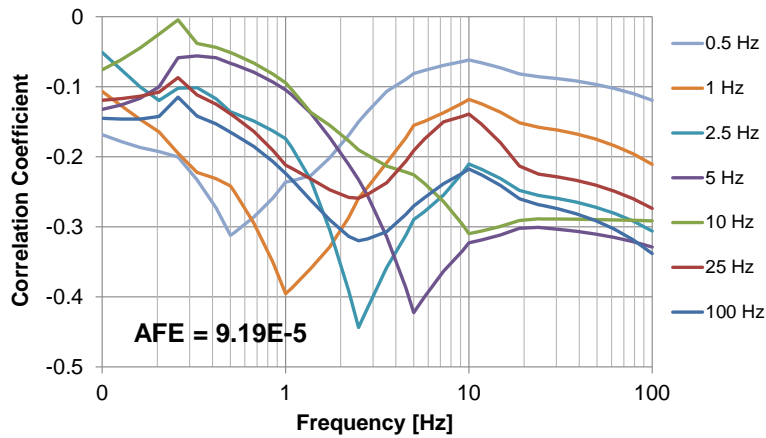


Figure 5. Correlation Coefficient between V/H ratio and horizontal motion for AFE of 9.19E-5

Following the procedures outlined in the previous section, the V/H ratios developed using the probabilistic approach for seven frequencies are presented in Figure 6. Also shown for comparison are the V/H ratio developed using deterministic approach and the V/H ratio developed using the probabilistic approach, but without consideration of correlation between V/H ratio and horizontal motion. The probabilistic V/H ratios developed without considering correlation are similar to the values from the deterministic approach, but more conservative for the high frequency range. A significant reduction in the calculated V/H ratios is observed for the probabilistic approach with consideration of the negative correlation between V/H ratio and horizontal motion.

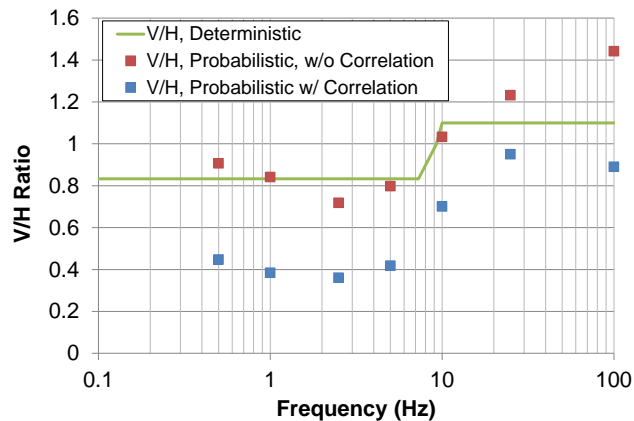


Figure 6. Comparison of V/H ratios from alternative models

The corresponding horizontal and vertical design spectra are presented in Figure 7. The vertical design spectrum developed using the proposed probabilistic model is lower than that developed using deterministic envelop approach. This difference is attributed to a better representation of uncertainties in the proposed model.

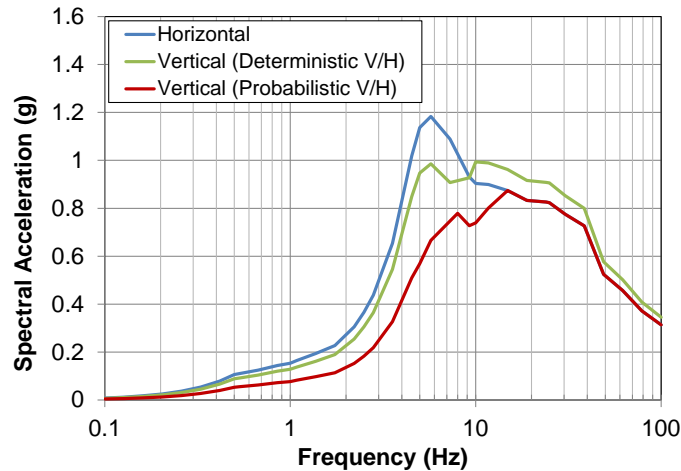


Figure 7. Horizontal design spectrum and vertical design spectra from alternative models

### Concluding Remarks

A probabilistic, site-specific and hazard-consistent V/H model is proposed in this study. This model is compatible with the approach to developing horizontal soil hazard. It is analogous to the convolution approach for site amplification factors as described in McGuire et al. (2001). It can be considered as an application in the framework of vector-valued PSHA (Bazzurro, 1998). Both epistemic and aleatory uncertainties can be incorporated in a natural manner as in a conventional PSHA. A case study shows that the proposed model may result in reduced vertical design spectra, and thus more realistic and economical design, compared with the commonly used deterministic envelop approach. The reduction in the vertical design spectrum is attributed to the improved treatment for the uncertainties and also the consideration of negative correlation between horizontal motion and V/H ratios. Currently, the application of the proposed model in the practice is subject to the scarcity of available standard deviation models (Bozorgnia and Campbell, 2004; Bommer et al., 2011; Edwards et al., 2011; Gülerce and Abrahamson, 2011). To the best of the authors' knowledge, only one correlation model (Gülerce and Abrahamson, 2011) is available in the literature. However, the correlation between H and V/H might be very stable as recent studies for the correlation of horizontal SAs (Baker and Jayaram, 2008; Jayaram et al., 2011) revealed that the correlation is in general dataset and GMPE independent. The proposed approach is useful for refining the vertical design spectra without conducting a formal vertical PSHA.

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