

## Dynamic Stiffness and Damping of Foundations for Jacket Structures

C. Latini<sup>1</sup>, V. Zania<sup>2</sup>, B. Johannesson<sup>3</sup>

### ABSTRACT

Foundation for offshore jacket structures may comprise of long floating piles. The dynamic response of floating piles (piles embedded in soil layers of height greater than the pile length) to horizontal load is herein investigated. The analytical solution of horizontally vibrating end bearing piles by Novak & Nogami (1977) has been modified. At first the soil resistance as defined by Nogami & Novak (1977) is determined, considering 3D wave propagation within linear soil layer with hysteretic damping. Thereafter, the dynamic response of the pile is estimated assuming soil pressure equal to the soil resistance and imposing displacement compatibility. A parametric study clarifies the role of the parameters involved i.e. the depth of the soil layer, the pile diameter and the soil layer shear wave velocity. Results are presented in terms of dimensionless graphs which highlight the frequency dependency of the dynamic stiffness and damping.

### Introduction

Nowadays, the offshore wind market is developing towards wind farms with higher capacity generators and in deeper waters, challenging the current offshore design procedures. So far the selection of the type of support structures for offshore wind turbines has been based on the water depth. In shallow waters, monopiles and monopod suction buckets are mostly utilized, while jacket structures with floating piles would be the design configuration for deeper waters following the traditional design of oil and gas industry (De Vries, 2007). In the design of offshore wind support structures fatigue derived from combined wind and wave loading is one of the critical issues. The potential of structural resonance with dynamic forces due to wind loading would result in large amplitude stresses and subsequent accelerated fatigue. For this reason the wind turbine support structure is practically designed by setting the tower fundamental resonance between the blade passing and the rotor frequency. In addition, the overall damping of the structure has an important impact on the fatigue damage, since the amplitude of vibrations at resonance is inversely proportional to the damping ratios (Devriendt et al., 2012).

Any structure subjected to dynamic load interacts with the foundation and the soil, altering thus the eigenfrequency and the damping (Kramer, 1996). Hence it is important to assess the dynamic stiffness and damping of the soil-foundation system. In order to rationally account for the dynamic interaction between the single pile foundation and the supporting soil deposit several analytical and numerical studies have been reported in the literature. Considering only those for

---

<sup>1</sup>Phd Student, Civil Engineering Department, Technical University of Denmark, Lyngby, Denmark, [chila@byg.dtu.dk](mailto:chila@byg.dtu.dk)

<sup>2</sup>Assistant Professor, Civil Engineering Department, Technical University of Denmark, Lyngby, Denmark, [vaza@byg.dtu.dk](mailto:vaza@byg.dtu.dk)

<sup>3</sup>Associate Professor, Civil Engineering Department, Technical University of Denmark, Lyngby, Denmark, [bjojo@byg.dtu.dk](mailto:bjojo@byg.dtu.dk)

linear elastic soil layer they can be categorized according to the following: a) rigorous analytical continuum solutions for end bearing piles (Novak & Nogami, 1977, Nogami & Novak, 1977, Zheng et al., 2013), where the soil is modelled as homogeneous layer with hysteretic material damping; b) Winkler type analytical solutions (Novak, 1974, Novak & Aboul-Ella, 1978, Mylonakis, 2001), where the supporting soil is replaced by a bed of independent elastic springs resting on a rigid base. For dynamic problems the use of Winkler foundation coefficients based on Baranov's equation for in plane and out plane vibration of a disk has been proposed by Novak (1974). An improved model incorporating in the analysis the normal and shear stresses acting on the upper and lower faces of a horizontal soil element by integrating the governing equations over the thickness of the soil layer has been developed by Mylonakis (2001); c) numerical continuum finite element solutions (Blaney et al., 1976, Roesset & Angelides, 1980, Velez et al., 1983, Gazetas, 1984, Gazetas & Dobry, 1984), where the soil is treated as an elastic continuum and the pile is assumed to have rigid cross section and it is modelled as series of regular beam segments. Very limited studies have investigated the response of floating piles either numerically (Gazetas & Dobry, 1984) or analytically (Nozoe et al., 1983). Hence the aim of this paper is to formulate an analytical solution for the dynamic response of floating piles focusing on the estimation of the dynamic stiffness and damping coefficients with respect to the frequency. Hence an appropriately modified formulation based on the rigorous analytical solution of soil-pile vibration by Novak & Nogami (1977) has been developed. The comparison of the end bearing with the floating pile is further discussed. A parametric study has been performed accounting for the effect of the soil profile, the pile diameter and the stiffness of the soil on the soil-pile system response.

### Methodology

The main assumptions of the solution presented here are: 1) the soil layer is linear, elastic, free at the surface; 2) the material damping is of the hysteretic type - frequency independent; 3) the pile is vertical, uniform, linearly elastic and of circular cross section. It is free at the tip and perfectly attached to the soil. In this formulation the reference system, RS1, is introduced to account for the fact that the height of the viscoelastic layer undergoing harmonic motion is larger than the pile length as shown in Figure 1.

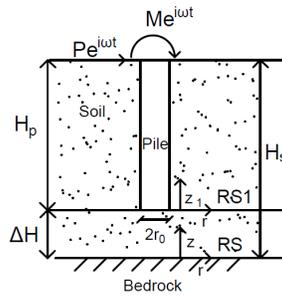


Figure 1. Analytical model of soil-pile system.

In the RS1 reference system the horizontal motion of the pile when subjected to harmonic excitation by end forces applied at the head of the pile is given as:

$$u(z_1, t) = u(z_1) e^{i\omega t} \quad (1)$$

where  $\omega$  is the circular frequency and  $z_1$  is the vertical coordinate of the pile. The governing equation of the pile motion follows the beam on elastic foundation by Hetényi (1971)

$$E_p I \frac{\partial^4}{\partial z_1^4} (ue^{i\omega t}) + m \frac{\partial^2}{\partial t^2} (ue^{i\omega t}) = -p(z_1)e^{i\omega t} \quad (2)$$

where  $E_p I$  is the bending stiffness of the pile,  $m$  is the mass of the pile per unit length and  $p(z_1)$  is the amplitude of the soil resistance to the motion of the pile. Whereas, the soil resistance expressed in the local pile's coordinate system is

$$p(z_1, t) = \sum_{n=1}^{\infty} \alpha_{hn} U_n \sin(h_n(z_1 + \Delta H)) \quad (3)$$

where  $\alpha_{hn}$  is the horizontal resistance factor depending on the pile radius  $r_0$ , shear modulus  $G$  and a number of dimensionless parameters such as the dimensionless frequency  $a_0 = H_s \omega / V_s$ , pile slenderness  $H_p / r_0$ , material hysteretic damping  $\zeta$  and Poisson's ratio  $\nu$ ;  $U_n$  is the modal amplitude independent of  $z$ ,  $\sin(h_n(z_1 + \Delta H))$  is the  $n^{\text{th}}$  mode shape of the soil layer,  $\Delta H = H_s - H_p$  and  $h_n = (\pi/2H_s)(2n - 1)$  where  $H_s$  is the depth of the soil layer and  $n$  is the mode number. Substituting Equation 3 into Equation 2 and eliminating the time variable,  $t$ , the following expression for the pile amplitude is obtained as:

$$E_p I \frac{d^4 u}{dz_1^4} - m \omega^2 u = - \sum_{n=1}^{\infty} \alpha_{hn} U_n \sin(h_n(z_1 + \Delta H)) \quad (4)$$

The solution to Equation 4 is given as a sum of the complete solution of the homogeneous equation  $u_h$ , and a particular solution of the non-homogeneous equation  $u_p$ . The particular solution  $u_p$  can be expressed as

$$u_p = \sum_{n=1}^{\infty} a_n \sin(h_n(z_1 + \Delta H)) \quad (5)$$

where  $a_n$  is a complex constant. Substitution of Equation 5 into Equation 4 yields

$$E_p I \sum_{n=1}^{\infty} a_n h_n^4 \sin(h_n(z_1 + \Delta H)) - m \omega^2 \sum_{n=1}^{\infty} a_n \sin(h_n(z_1 + \Delta H)) = - \sum_{n=1}^{\infty} \alpha_{hn} U_n \sin(h_n(z_1 + \Delta H)) \quad (6)$$

Hence, the constant  $a_n$  can be determined as

$$a_n = \frac{-\alpha_{hn} U_n}{E_p I h_n^4 - m \omega^2} \quad (7)$$

The solution of the homogeneous equation can be written as

$$u_h = A \sin(\lambda z_1) + B \cos(\lambda z_1) + C \sinh(\lambda z_1) + D \cosh(\lambda z_1) \quad (8)$$

where  $A$ ,  $B$ ,  $C$  and  $D$  are the integration constants obtained by the boundary conditions at the tip of the pile and

$$\lambda = \sqrt[4]{\frac{m \omega^2}{E_p I}} \quad (9)$$

Then the pile displacement is given as:

$$u(z_1) = A \sin(\lambda z_1) + B \cos(\lambda z_1) + C \sinh(\lambda z_1) + D \cosh(\lambda z_1) - \sum_{n=1}^{\infty} \frac{\alpha_{hn} U_n}{E_p I h_n^4 - m \omega^2} \sin(h_n(z_1 + \Delta H)) \quad (10)$$

The displacement of the soil layer at the pile can be expressed as

$$U(z_1) = \sum_{n=1}^{\infty} U_n \sin(h_n(z_1 + \Delta H)) \quad (11)$$

The displacement compatibility between the pile and the soil layer is imposed. Then, the variable  $z_1$  is written as  $z_1 = z - \Delta H$  and expanding  $\sin(\lambda(z - \Delta H))$ ,  $\cos(\lambda(z - \Delta H))$ ,  $\sinh(\lambda(z - \Delta H))$  and  $\cosh(\lambda(z - \Delta H))$  into a Fourier sine series of argument  $h_n z$ , the following formula is obtained:

$$U_n = \frac{AF_{1n} + BF_{2n} + CF_{3n} + DF_{4n}}{1 + \left[ \frac{\alpha_{hn}}{E_p I h_n^4 - m \omega^2} \right]} \quad (12)$$

where

$$\begin{cases} F_{1n} = \frac{2}{H_s} \int_0^{H_s} \sin(\lambda(z - \Delta H)) \sin(h_n z) dz \\ F_{2n} = \frac{2}{H_s} \int_0^{H_s} \cos(\lambda(z - \Delta H)) \sin(h_n z) dz \\ F_{3n} = \frac{2}{H_s} \int_0^{H_s} \sinh(\lambda(z - \Delta H)) \sin(h_n z) dz \\ F_{4n} = \frac{2}{H_s} \int_0^{H_s} \cosh(\lambda(z - \Delta H)) \sin(h_n z) dz \end{cases} \quad (13)$$

Substituting  $U_n$  into Equation 10, the amplitude of the pile motion is

$$u(z) = A \sin(\lambda(z - \Delta H)) + B \cos(\lambda(z - \Delta H)) + C \sinh(\lambda(z - \Delta H)) + D \cosh(\lambda(z - \Delta H)) - \sum_{n=1}^{\infty} \frac{\alpha_{hn} (AF_{1n} + BF_{2n} + CF_{3n} + DF_{4n})}{E_p I h_n^4 - m \omega^2 + \alpha_{hn}} \sin h_n z \quad (14)$$

Using the displacement of the pile presented in Equation 14, the amplitude of the angle of rotation,  $\theta$ , the bending moment,  $M$ , and the shear force,  $S$ , are obtained from the corresponding derivatives. The unknown coefficients  $A$ ,  $B$ ,  $C$ ,  $D$  are estimated by considering the boundary conditions and applying a unit horizontal translation and a unit rotation at the pile head. The dynamic impedances  $K_{su}$ ,  $K_{s\theta}$ ,  $K_{mu}$  and  $K_{m\theta}$  at the level of the pile head are then calculated as shear forces,  $S$ , and moments,  $M$ , for unit displacement,  $u$ , and rotation,  $\theta$ .

## Parametric Study

The dynamic response of floating piles is analyzed by employing the method described in the previous section. In the current study the comparison of the end bearing with the floating pile is investigated and further, the effect of the pile diameter and the shear wave velocity of the soil layer on the soil-floating pile response are explored. This leads to some considerations of the role of popular dimensionless parameters such as the stiffness ratio  $E_p/E_s$  and the pile flexibility factor  $K_r$  (Poulos & Davis, 1980), on the dynamic components of the stiffness and the damping. The rationale for the selection of the dimensionless parameters was to examine small diameter ( $d = 1 \div 3m$ ) – for offshore applications - hollow, flexible, steel piles embedded in a homogeneous soil layer with constant profile of shear wave velocities ( $V_s = 100 \div 400m/s$ ), pile's thickness ( $t = r_0/50$ ), hysteretic material damping ( $\xi = 2.5\%$ ) and Poisson's ratio ( $\nu =$

0.35) at quite wide frequency range including at least the third eigenfrequency of the soil layer ( $\alpha_0 = 5/2\pi$ ). The reference case analyzed is  $d = 1\text{m}$ ,  $V_s = 250\text{m/s}$ ,  $H_p = 20\text{m}$  and  $H_s = 30\text{m}$ . Note that all the investigated cases resemble flexible pile response according to the flexibility criterion suggested by Poulos and Davis (1980).

### Comparison with end bearing piles

The comparison between horizontally vibrating end bearing piles and floating piles is presented. The reference case is analyzed by varying the boundary conditions at the pile tip, fixed and hinged for end bearing pile and, hinged and free for floating piles. In Figure 2a the dynamic component (real part of the complex valued stiffness terms divided by the corresponding static component  $K_{xx}^0$ ) of the three stiffness terms is presented with respect to the non-dimensional frequency. It is recorded a drop of stiffness at the first eigenfrequency of the soil layer ( $\alpha_0 = \pi/2$ ), which is more marked in the case of end bearing piles, while the drop of stiffness is observed at all three eigenfrequencies for the free tip floating pile. Sensitivity of the dynamic stiffness on the boundary conditions at the pile tip is observed only in the case of floating piles. In Figure 2b the dynamic component (imaginary part of the complex valued stiffness terms divided by the corresponding dynamic component  $K_{xx}$ ) of the three stiffness terms is shown with respect to the non-dimensional frequency.

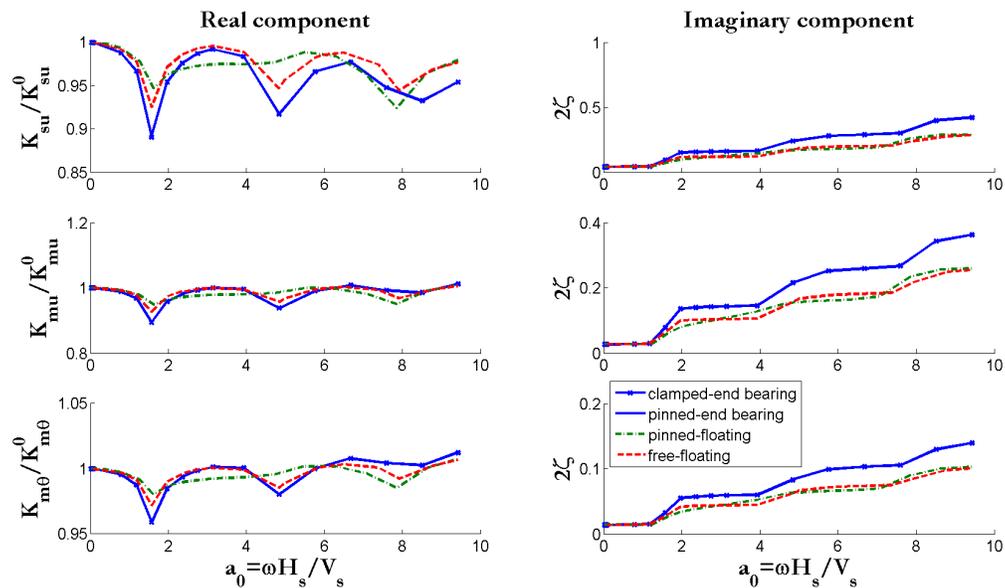


Figure 2. Variation of the three dynamic stiffness coefficients with respect to the dimensionless frequency. The real component (a) and the imaginary component (b) for the reference case and two soil profiles with various boundary conditions.

The radiation damping (viscous type) is generated for frequencies higher than the first eigenfrequency of the soil layer. After that, its trend increases almost monotonously over the frequency range for the case of the end bearing piles, while the pattern is less steep for floating piles. A slight change in the slope of the damping is also marked after each eigenfrequency of the soil layer. In Figure 3 the modal displacement of the pile and the soil layer at the eigenfrequencies of the soil layer (1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup>) are illustrated along the depth for both the

floating (Figure 3a) and the end bearing pile (Figure 3b). It seems like that the floating pile allows for the development of the 3<sup>rd</sup> eigenfrequency. At the higher modes the modal response of the floating pile appears closer to the one of the soil layer alone.

### Effect of the pile diameter

In Figure 4 the effect of the pile diameter is illustrated on the dynamic impedances by considering all the other parameters identical to the reference case.

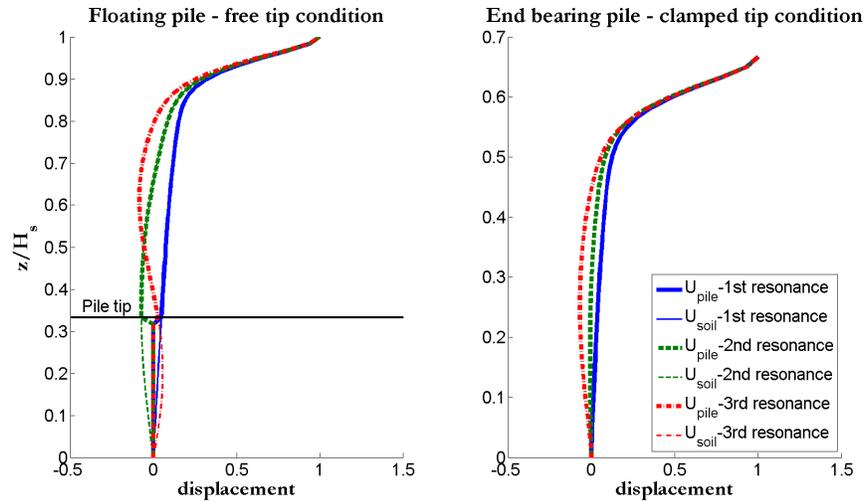


Figure 3. Distribution of the soil and pile displacement along the normalized depth  $z/H_s$  at the three first eigenfrequencies of the soil layer. The floating pile (a) and the end bearing pile (b) for the reference case are shown.

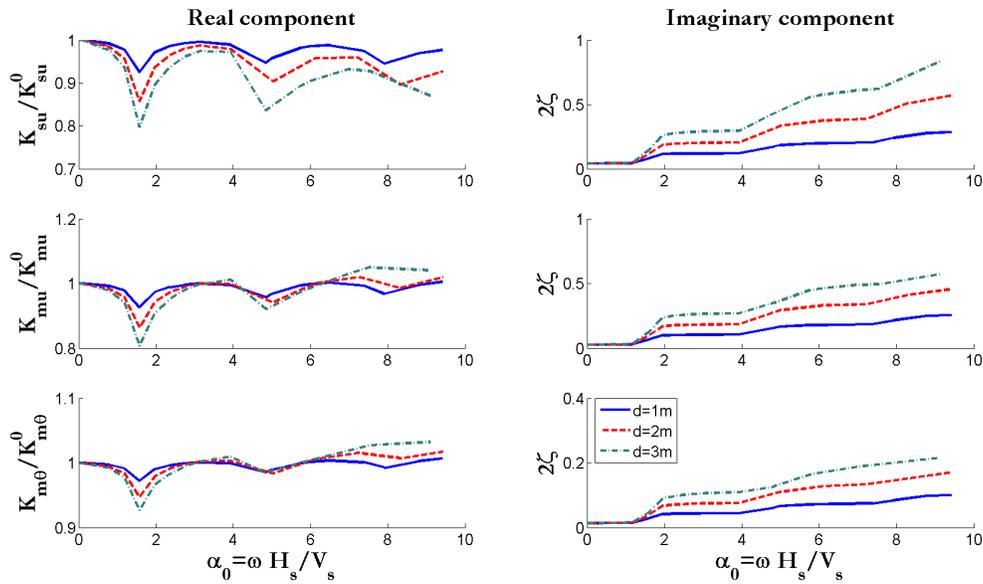


Figure 4. Variation of the three dynamic stiffness coefficients with respect to the dimensionless frequency. Effect of the diameter on the real component (a) and the imaginary component (b).

By keeping unchanged the height of the soil layer and the pile length the dimensionless parameters  $H_p/d$  and  $K_r$  varied. By decreasing the pile diameter the drop of stiffness at the resonance with the soil layer becomes less remarkable and a smoother pattern of the dynamic stiffness is obtained. The effect of the diameter is more prominent for the translational component of the dynamic stiffness, where the increase of the diameter enhances the dynamic stiffness reduction. The imaginary part of the dynamic component is shown in Figure 4b. The radiation damping exhibits almost constant variation in the intermediate frequency interval ( $\alpha_0 = 2 - 4$ ) and therefore, it can be roughly approximated by linear function in the high frequency range. Moreover, it is observed that the radiation damping rises by increasing the pile diameter. This suggests that an increase of the pile flexibility factor  $K_r$  and a decrease of the slenderness ratio  $H_p/d$  determine greater values of the imaginary and smaller value of the real component of the dynamic impedances.

### Effect of the soil stiffness

In Figure 5a the real part of dynamic impedances is shown for different values of the shear wave velocity of the soil layer. By keeping the same values as in the reference case for the height of the pile and the soil layer the dimensionless parameters  $K_r$  and  $E_p/E_s$  varied.

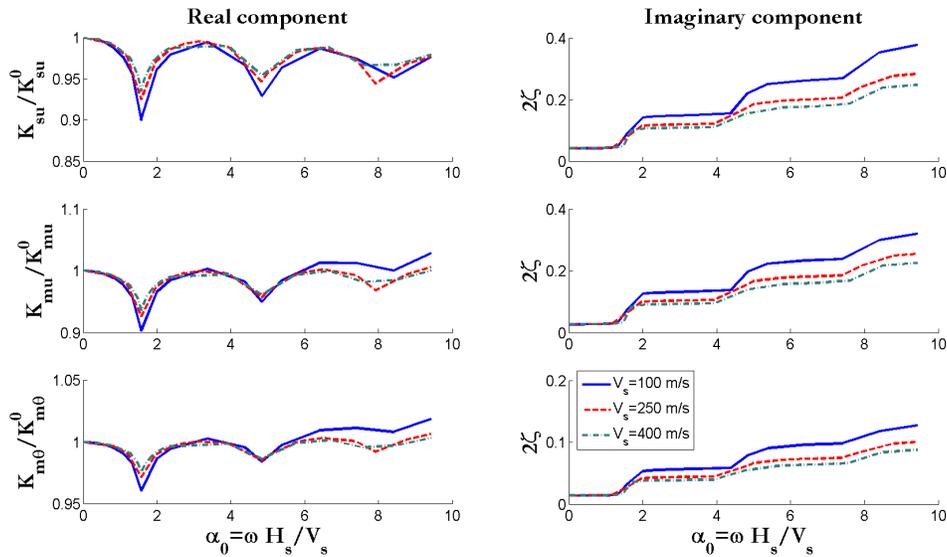


Figure 5. Variation of the three dynamic stiffness coefficients with respect to the dimensionless frequency. Effect of  $V_s$  on the real component (a) and the imaginary component (b).

Slightly scattered results are obtained by decreasing the shear wave velocity of the soil layer. This implies that the effect of the dimensionless parameters  $K_r$  and  $E_p/E_s$  is less prominent. In addition, the drop of stiffness recorded at the first eigenfrequency of the soil layer is slightly more marked for soft soil profiles ( $V_s=100\text{m/s}$ ). Moreover, it is noticed that the cross coupling and rocking stiffness coefficients exhibit higher values than the corresponding static component at higher frequencies. In Figure 5b the imaginary component is illustrated for different values of the shear wave velocity of the soil layer. A flat trend is observed at the intermediate frequencies ( $\alpha_0 = 2 - 4$ ), while it increases monotonously in the high frequency range. By increasing  $E_p/E_s$ , the damping increases, an observation consistent to flexible end bearing pile's response.

## Conclusions

The analytical solution for horizontally vibrating end bearing piles by Novak & Nogami (1977) has been modified for floating piles. The comparison between horizontally vibrating end bearing piles and floating piles has indicated that the drop of stiffness is stronger in the case of end bearing piles, while the radiation damping is suppressed for floating piles. The results of small diameter flexible floating piles have shown that the dynamic impedances are significantly affected by the variation of the pile diameter, whereas they are only slightly sensitive on the shear wave velocity.

## Acknowledgments

This work has been supported by the Danish Council for Strategic Research through the project “Advancing Beyond Shallow waterS (ABYSS) - Optimal design of offshore wind turbine support structures”.

## References

- Blaney GW, Kausel E, Roesset J. Dynamic stiffness of piles, Proceedings of 2nd International Conference on Numerical Methods in Geomechanics, Blacksburg, USA, 2, 1001–1012, 1976.
- De Vries WE. Assessment of bottom-mounted support structure type with conventional design stiffness and installation techniques for typical deep water sites. Derivable report 2007.
- Devriendt C, Jordaens PJ, De Sitter G, Guillaume P. Estimating damping of an offshore wind turbine using n overspeed stop and ambient excitation. 15th International Conference on Experimental Mechanics 2012; 2897.
- Gazetas G, Dobry R. Horizontal response of piles in layered soils. *Journal of Geotechnical Engineering* 1984; **110**(1): 20-40.
- Gazetas G. Seismic response of end-bearing single piles. *Soil Dynamics and Earthquake Engineering* 1984; **3**(2): 82–93.
- Hetényi M. *Beams on Elastic Foundation - Theory with Applications in the Fields of Civil and Mechanical Engineering*. University of Michigan Press 1971.
- Kramer SL. *Geotechnical Earthquake engineering*, Prentice Hall, New Jersey, U.S.A, 1996.
- Mylonakis G. Elastodynamic model for large diameter end-bearing shafts. *Soils and foundations* 2001; **41**(3): 31-44.
- Nogami T, Novak M. Resistance of soil to a horizontally vibrating pile. *Earthquake Engineering and Structural Dynamics* 1977; **5**(3): 249-261.
- Novak M, Aboul- Ella F. Impedance functions of piles in layered media. *Journal of Engineering Mechanics Division* 1978; **104**(6): 643-661.
- Novak M, Nogami T. Soil-pile interaction in horizontal vibration. *Earthquake engineering and structural dynamics* 1977; **5**: 263-281.
- Novak M. Dynamic stiffness and damping of piles. *Canadian Geotechnical Journal* 1974; **11**: 574-598.
- Nozoe H, Gyōten Y, Fukusumi T. Dynamic analysis of a soil-pile system by the finite Fourier-Henkel transformation method – Case of a floating pile in horizontal vibration. *Theoretical and applied mechanics* 1985, **33**: 377-392.
- Poulos HG, Davis EH. *Pile foundation analysis and design*. New York, NY: John Wiley & Sons; 1980.
- Roesset JM, Angelides D. Dynamic stiffness of piles. Proceedings of the International Conference on Numerical Methods in Offshore Piling, Institution of Civil Engineers (ICE), London, 1980.
- Velez A, Gazetas G, Krishnan R. Lateral Dynamic Response of Constrained Head Piles. *Journal of Geotechnical*

*Engineering* 1983; **109**(8): 1063–1081.

Zheng C, Liu H, Ding X, Fu Q. Horizontal Vibration of a Large-Diameter Pipe Pile in Viscoelastic Soil, *Mathematical Problems in Engineering* 2013.