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Strain Space Multiple Mechanism Model for Clay Under Monotonic and Cyclic Loads

S. Iai[1], K. Ueda[2], O. Ozutsumi[3]

ABSTRACT

The strain space multiple mechanism model, originally developed for cyclic behavior of granular materials such as sand, is adapted for idealizing stress strain behavior of clay under monotonic and cyclic loads. The proposed model has advantages over the conventional elasto-plastic models of Cam-clay type in the facts that (1) arbitrary initial K_0 state can be analyzed by static gravity analysis, (2) stress induced anisotropy (i.e. effect of initial shear) in the steady (critical) state can be analyzed based on Shibata's dilatancy model (1963), (3) over-consolidated clay can be analyzed by defining the dilatancy at the steady state based on over-consolidation ratio, (4) strain rate effects for monotonic and cyclic shear can be analyzed based on Isotach/TESRA models proposed by Tatsuoka et al (2002) in the strain rate ranging from zero to infinity in addition to the conventional strain rate effects of secondary consolidation (creep) type. Performance of the proposed model is demonstrated through simulation of drained/undrained behavior of clay under monotonic and cyclic loading.

Introduction

The mini-symposium, entitled Combined Geotechnical Hazards, provides a forum on new and challenging issues, including combined failure due to Tsunami after earthquake, effect of aftershock on liquefaction after mainshock, effects of deformation due to consolidation of clay before an earthquake, and complex soil-structure-foundation-water interaction during earthquakes. These issues have not been studied in the past but have been exposed by recent case histories during earthquakes as new engineering problems to be solved by advanced and sophisticated approaches.

As one of the papers prepared for the mini-symposium for discussing the complex effects, this paper presents formulation and fundamental performance of a new constitutive model for clay based on the framework of the strain space multiple mechanism model of granular materials. Although numerous elasto-plastic models of conventional Cam-clay type have been proposed for clay over years, application of these models to seismic response of clayey ground or clay and sand mixtures in engineering practice is none or extremely limited to the knowledge of the authors. Major drawbacks of these conventional elasto-plastic (or visco-elasto-plastic) models for clay are: (1) initial K_0 state which has significant effects on deformation due to seismic load is specified as soil parameters, not analyzed through static gravity analysis for representing

[1]Professor, Disaster Prevention Research Institute, Kyoto University, Kyoto, Japan, iai.susumu.6x@kyoto-u.ac.jp
[2]Researcher, Railway Technical Research Institute, Tokyo, Japan, ueda@rtri.or.jp
[3]President, Meisosha Co., Tokyo, ozutsumi@meisosha.co.jp

realistic initial stress field in soil-structure systems, (2) viscous effects of clay is typically formulated for simulating the secondary consolidation behavior and not for earthquake loading which may include extremely high strain rate in shear strain. This study aims to solve these problems through the framework of strain space multiple mechanism model.

Strain Space Multiple Mechanism Model and K_0 State Analysis by Gravity

The basic form of the strain space multiple mechanism model for sand is given in two dimension as a relation between macroscopic stress tensor $\boldsymbol{\sigma}$ and strain tensor $\boldsymbol{\varepsilon}$ by (Iai et al. 2011)

$$\boldsymbol{\sigma} = -p\mathbf{I} + \int_0^{\pi} q\langle \mathbf{t}\otimes\mathbf{n}\rangle \mathrm{d}\omega,\ \langle \mathbf{t}\otimes\mathbf{n}\rangle = \mathbf{t}\otimes\mathbf{n} + \mathbf{n}\otimes\mathbf{t} \tag{1}$$

where $\mathbf{I}$ denotes identity tensor, $\mathbf{n}$ and $\mathbf{t}$ denote the direction vectors normal and tangential to particle contacts within granular materials, and the scalars p and q are functions with history of volumetric strain ε, volumetric strain due to dilatancy ε_{d}, and virtual simple shear strain γ by

$$p = p(\varepsilon'),\ q = q(\gamma),\ \varepsilon' = \varepsilon - \varepsilon_{\mathrm{d}},\ \varepsilon = \mathbf{I}:\boldsymbol{\varepsilon},\ \gamma = \langle \mathbf{t}\otimes\mathbf{n}\rangle : \boldsymbol{\varepsilon} \tag{2}$$

In particular, q is given as hysteretic hyperbolic function and approximates the hysteretic hyperbolic curve of macroscopic shear stress and shear strain in cyclic simple shear. Tangential bulk modulus and elastic shear modulus are given with power functions of a confining pressure p from the values at reference state $p = p_{\mathrm{a}}$ as

$$K = K_{\mathrm{La}}\left(p/p_{\mathrm{a}}\right)^{n_K},\ G = G_{\mathrm{ma}}\left(p/p_{\mathrm{a}}\right)^{m_G} \tag{3}$$

Under the condition of K_0 consolidation where lateral normal strain is fixed, vertical normal strain governs both volumetric and deviator strains. Using the same power index $n_K = m_G = m$, this K_0 condition leads to the following equation which relates a coefficient of earth pressure at rest K_0 with an internal friction angle ϕ_{f}, as

$$K_{\mathrm{La}} = \frac{1}{1-m}\frac{(1+\sin\phi_{\mathrm{f}})K_0 - (1-\sin\phi_{\mathrm{f}})}{(1-K_0)\sin\phi_{\mathrm{f}}}G_{\mathrm{ma}} \tag{4}$$

In practice of analysis, the power index can be set $m = 0.5$ (or any number except for 1) for static gravity analysis for achieving realistic K_0 state for level ground.

Steady State and Stress Induced Anisotropy

In the strain space multiple mechanism model, steady state line in p-ε plane is defined by a vertical shift of a normal consolidation curve as shown in Figure 1. The normal consolidation curve for analysis of clay is given with a reduction factor r_K with the power index $l_K = l_G = 1$ in

$$K_{\mathrm{L/U}} = r_K K_{\mathrm{U0}}\left(p/p_0\right)^{l_K},\ G = G_{\mathrm{m0}}\left(p/p_0\right)^{l_G} \tag{5}$$

where the subscript 0 implies initial state for consolidation or dynamic analysis after static gravity analysis for representing the initial stress-strain field.

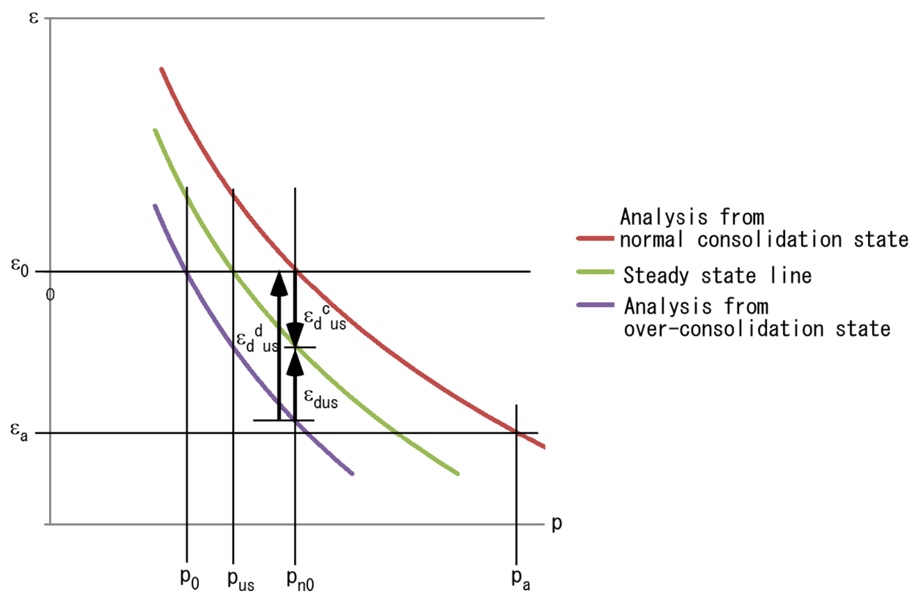


Figure 1. Steady state line

Integrated form of volumetric mechanism in Equation 5 with $l_K = 1$ is given as a typical straight line in e-ln$p$ plane. The vertical shift of the curve is measured by the contractive component of dilatancy at the steady state $\varepsilon_{\mathrm{d\,us}}^{\mathrm{c}}$ where the subscript US implies the steady (ultimate) state. This component is defined based on Shibata's dilatancy model (Shibata 1963) as

$$\varepsilon_{\mathrm{d}}^{\mathrm{c}} = -D\eta^{*}, \text{ where } \eta^{*} = \left(1/\sqrt{2}\right)\left|\bar{\boldsymbol{\sigma}} / p - \bar{\boldsymbol{\sigma}}_0 / p_0\right|, \ \bar{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - p\mathbf{I} \tag{6}$$

where the initial stress ratio term $\bar{\boldsymbol{\sigma}}_0 / p_0$ represents the stress induced anisotropy (Sekiguchi and Ohta 1977). The parameter D is related with the parameters for Cam-clay model M, λ, κ, and initial void ratio e_0 as (Iizuka and Ohta 1987)

$$D\mathrm{M} = \left(1 - \kappa / \lambda\right)\varepsilon_{\mathrm{m0}} \quad , \quad \varepsilon_{\mathrm{m0}} = \lambda / \left(1 + e_0\right) = p_0 / \left(r_K K_{\mathrm{U0}}\right) \tag{7}$$

Consequently, the contractive component of dilatancy at steady state is given by the following equation and is substituted for the upper limit of the contractive component of dilatancy $\varepsilon_{\mathrm{d}}^{\mathrm{cm}}$ in the strain space multiple mechanism model.

$$\varepsilon_{\mathrm{d\,us}}^{\mathrm{c}} = -\left(1 - r_{K_{\mathrm{us}}}\right)\left(\eta^{*}{}_{\mathrm{us}} / \mathrm{M}\right)\varepsilon_{\mathrm{m0}}, \text{ where } r_{K_{\mathrm{us}}} = \kappa / \lambda \tag{8}$$

For the analysis of over-consolidated clay, the curve for governing the volumetric mechanism is defined by vertical shift of normal consolidation curve. The initial confining pressure p_0 at the over-consolidated state and the corresponding confining pressure at the normal consolidation line p_{n0} is defined by a parameter as $p_{\mathrm{n0}} = r_{p_{\mathrm{n0}}} p_0$. Then the dilatancy at the steady state is given by

$$\varepsilon_{\mathrm{d\,us}} = \varepsilon_{\mathrm{d\,us}}^{\mathrm{c}} + \varepsilon_{\mathrm{d\,us}}^{\mathrm{d}}, \ \varepsilon_{\mathrm{d\,us}}^{\mathrm{d}} = \ln\left(p_{\mathrm{n0}} / p_0\right)\varepsilon_{\mathrm{m0}} \tag{9}$$

In addition, a parameter of cohesion c_a at reference state is introduced for defining shear strength in over-consolidation regime on behalf of Hvorslev line.

The model parameters described above are summarized in Table 1. Many of the parameters for modeling of sand behavior are no longer required for modeling of clay and fixed to the default values: i.e. the power index $l_K = l_G = 1$, phase transformation angle $\phi_p = \phi_f$, a parameter for dilatancy $r_{\varepsilon_d} = 1$, a parameter for initial phase of dilatancy q_1=1, upper limit for contractive dilatancy $\varepsilon_d^{cm} = \varepsilon_{dus}^{c}$ (by substitution as part of algorithm), a small positive number for numerical stability at liquefaction S_1=0.005, a parameter for defining elastic range for liquefaction c_1=1. Steady state shear strength q_{us} is not used and replaced by the parameter $r_{Kus} = \kappa / \lambda$ for controlling dilatancy at the steady state.

Table 1. Model parameters.

Symbol	Mechanism	Parameter designation
$K_{L/Ua}$	Volumetric	Bulk modulus at reference state
n_K, m_G	Volumetric/Shear	Power index for bulk/shear moduli at static gravity analysis
r_K	Volumetric	Reduction factor of bulk modulus for consolidation analysis
$r_{p_{n0}}$	Volumetric	Over-consolidation state $r_{p_{n0}} = p_{n0} / p_0 = \mathrm{OCR}^{(1-\kappa/\lambda)}$
G_{ma}	Shear	Shear modulus at reference state
ϕ_f	Shear	Internal friction angle (M=sinϕ_f)
c_a	Shear	Cohesion at reference state (for over-consolidation regime)
h_{max}	Shear	Upper bound for hysteretic damping factor
$r_{\varepsilon_d^c}$	Dilatancy	Parameter controlling contractive component
q_2	Dilatancy	Parameter controlling later phase of contractive component
r_{Kus}	Dilatancy	Parameter controlling dilatancy at the steady state

Model Performance Under Drained/Undrained Monotonic Loadings

Performance of the proposed model is studied for its capability to simulate stress strain behavior under drained monotonic loading. The parameters used for this study are shown in Table 2. A clay element is initially consolidated at K_0=0.6, and then strain controlled tests are performed under various strain paths with volumetric strain dominant regime (Figure 2(a)) and deviator strain dominant regime (Figure 3(a)). The computed results in the stress paths and the consolidation curves are shown in Figures 2(b)&(c) and 3(b)&(c).

Primary findings on the performance of the proposed model are:

(1) After K_0 consolidation, the stress path following the initial K_0 line is given by the strain path with volumetric strain increase with constant deviator strain.

(2) After K_0 consolidation, the stress paths given by the strain paths with fixed lateral normal strain (i.e. $\varepsilon_x - \varepsilon_y = -(\varepsilon_x + \varepsilon_y)$) are directed closer to the shear failure line.

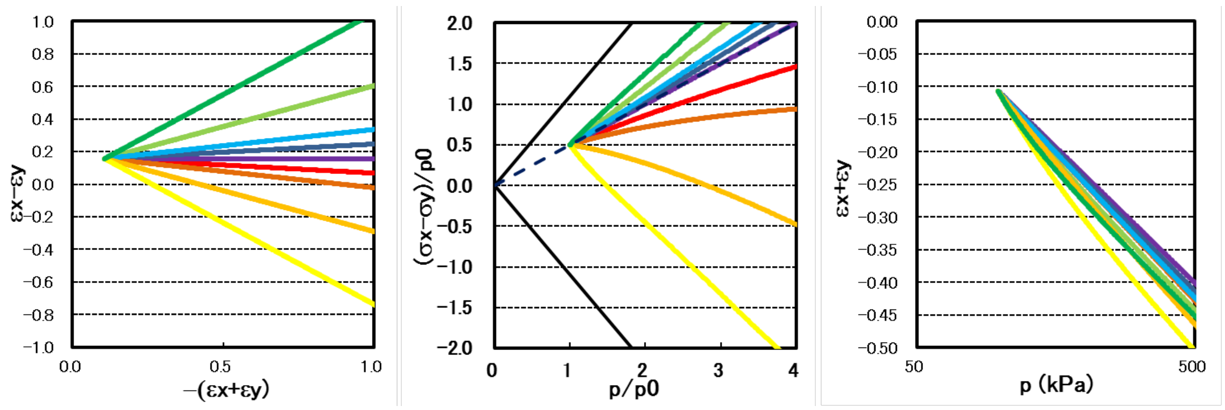
(3) After K_0 consolidation, the consolidation curves approaches steady state lines affected by stress induced anisotropy.

(4) The proposed model does not exhibit the phenomenon of “Meta-stability” (Takeyama et al.

2013) that traps a certain range of stress paths into a single stress path on the same K_0 line.

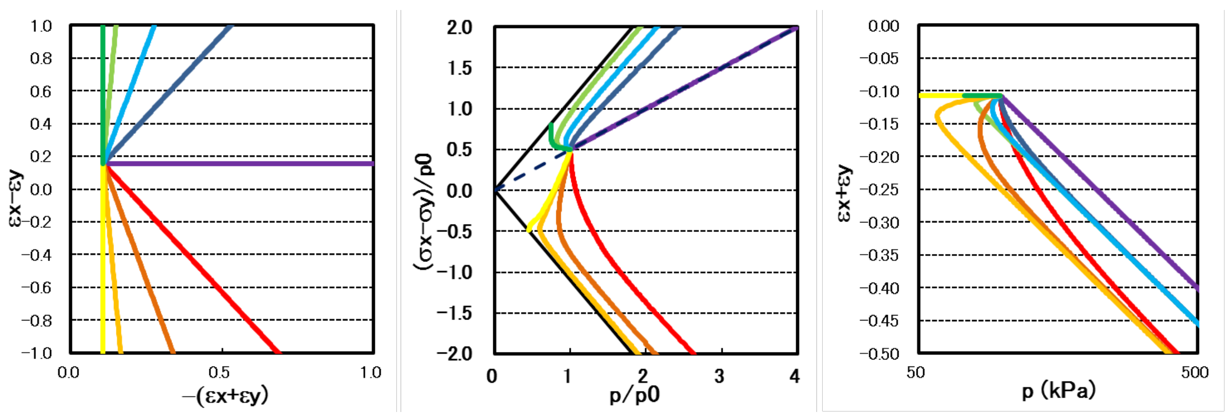
Table 2. Model parameters for simulation ($p_{\mathrm{a}} = 135.5\mathrm{kPa}$).

$K_{\mathrm{L/Ua}}$	m	r_K	G_{ma}	ϕ_{f}	c_{a}	h_{max}	$r_{\varepsilon_{\mathrm{d}}^{\mathrm{c}}}$	q_2	$r_{K\mathrm{us}}$
2155 kPa	0.5	0.348	352 kPa	33.3°	20 kPa	0.24	0.85	0	0.456



(a) Strain paths (b) Computed stress paths (c) Computed consolidation curves

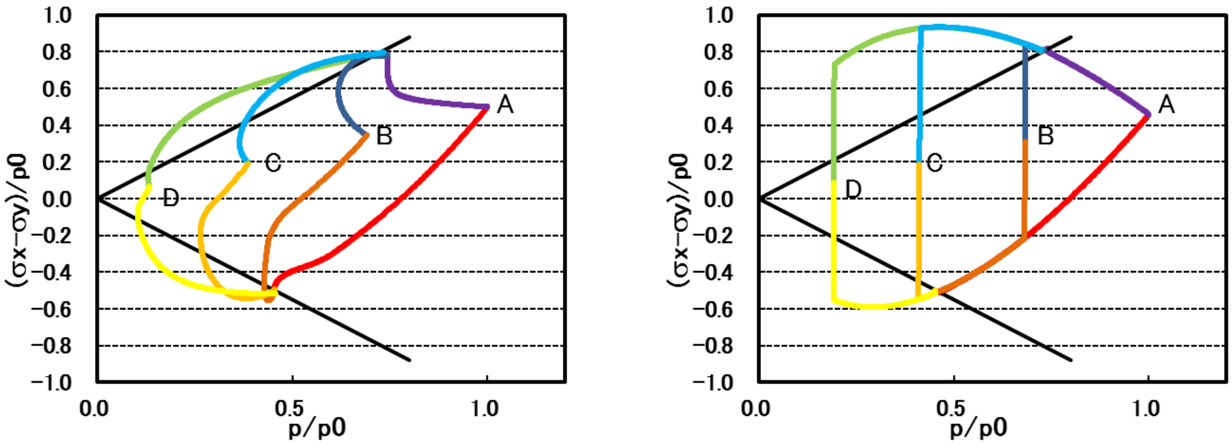
Figure 2 Performance under drained monotonic loading (for volumetric strain dominant regime)



(a) Strain paths (b) Computed stress paths (c) Computed consolidation curves

Figure 3 Performance under drained monotonic loading (for deviator strain dominant regime)

Performance of the proposed model is also studied for its capability to simulate stress strain behavior under undrained monotonic loading. The same parameters shown in Table 2 are used for the simulation. A clay element is initially consolidated at K_0=0.6 with over-consolidation ratios of OCR=1, 2, 5, and 20 to achieve the same initial void ratio of e_0=2.3, and then sheared in compression and extension. The computed results shown in Figure 4 indicate that the proposed model reasonably simulates the steady state strengths affected by stress induced anisotropy and also reasonably simulate the contractive and dilative behavior in the over-consolidated regime.



(a) Proposed model

(b) Sekiguchi-Ohta model (elasto-plastic model)

Figure 4 Performance under undrained shear (A, B, C, D corresponds to OCR=1, 2, 5, 20)

Strain Rate Effects

Strain rate effects in the proposed model are formulated in the strain space multiple mechanism model with Isotach/TESRA mechanism for representing viscosity during shear (Tatsuoka et al. 2002) in addition to the conventional creep mechanism for representing secondary consolidation. The constitutive equation is written by adding the viscous terms $\boldsymbol{\sigma}^*$ to Equation 1 as

$$\boldsymbol{\sigma} = -p\mathbf{I} + \int_0^{\pi} q\left\langle \mathbf{t} \otimes \mathbf{n} \right\rangle \mathrm{d}\omega + \boldsymbol{\sigma}^* \tag{10}$$

The conventional creep mechanism for representing secondary consolidation is written by adding the volumetric strain due to creep $\varepsilon_{\mathrm{c}} = -\alpha_{\mathrm{c}}\left(\ln\left(t + t_{\mathrm{c}}\right) - \ln t_{\mathrm{c}}\right)$ as $\varepsilon' = \varepsilon - \varepsilon_{\mathrm{d}} - \varepsilon_{\mathrm{c}}$ in Equation 2, where the term $t_{\mathrm{c}} = \alpha_{\mathrm{c}} / \dot{v}_0$ is given from secondary consolidation coefficient α_{c} and initial volumetric creep strain rate $\dot{v}_0 \left(= -\dot{\varepsilon}_{\mathrm{c}0}\right)$.

The viscous terms $\boldsymbol{\sigma}^*$ for representing shear strain rate effects Equation 10 is given by using the decay parameter for TESRA r_{TSR} as the following recurrence equation from the step p to the current step (without superscript p+1):

$$\boldsymbol{\sigma}^* = \boldsymbol{\sigma}^{*p} \exp\left(\Delta\overline{\gamma} \ln r_{\mathrm{TSR}}\right) + \Delta\boldsymbol{\sigma}^*,\ \Delta\overline{\gamma} = \sqrt{2}\left|\overline{\boldsymbol{\varepsilon}} - \overline{\boldsymbol{\varepsilon}}^p\right|,\ \overline{\boldsymbol{\varepsilon}} = \boldsymbol{\varepsilon} - \left(1/2\right)\varepsilon\mathbf{I} \tag{11}$$

$$\Delta\boldsymbol{\sigma}^* = \left[\int_0^{\pi} \Delta q^* \left\langle \mathbf{t} \otimes \mathbf{n} \right\rangle \mathrm{d}\omega\right] \exp\left(0.5\Delta\overline{\gamma} \ln r_{\mathrm{TSR}}\right),\ \Delta q^* = q^* - q^{*p} \tag{12}$$

The incremental term of viscous effect $\Delta\boldsymbol{\sigma}^*$ is formulated in the strain space multiple mechanism model by regarding the viscosity function of Isotach model as the upper limit of linear viscous damping at the given strain rate regime. This formulation has advantage in stability in numerical analysis of clayey ground during earthquakes over the original formulation of Isotach/TESRA model by Tatsuoka et al (2002). The formulation is written as

$$q^* = r_{\mathrm{Iso}} r_q \frac{\dot{\gamma}^*}{\max\left|\dot{\gamma}^*\right|} \left(q_{\mathrm{v}} \frac{\left|\xi - \xi_{\mathrm{r}}\right|}{1 + \left|\xi - \xi_{\mathrm{r}}\right|} \right),\ r_q = 1 - \exp\left(1 - \left(1 + \max\left|\dot{\gamma}^*\right|\right)^{q_6}\right) \tag{13}$$

where $\xi = \gamma / \gamma_{\mathrm{v}}$, $\dot{\gamma}^* = r_{\dot{\gamma}} \dot{\gamma}$, and subscript "r" implies strain reversal from loading to unloading. Isotach/TESRA parameters used by Tatsuoka et al (2002) are replaced by $r_{\mathrm{TSR}} = r_1$, $r_{\mathrm{Iso}} = \alpha$, $r_{\dot{\gamma}} = 1/\dot{\varepsilon}_{\mathrm{r}}^{\mathrm{ir}}$, $q_6 = m$. Shear strength q_{v} and reference strain γ_{v} in virtual simple shear mechanism are given by macroscopic shear strength and reference strain as $q_{\mathrm{v}} = \tau_{\mathrm{m}} / 2$ and $\gamma_{\mathrm{v}} = (\pi / 4)\gamma_{\mathrm{m}}$. The model parameters for strain rate effects are summarized in Table 3. The specific characteristics of Isotach/TESRA model relative to the simple linear viscous damping are in the fact that the parameter q_6 is typically very small so that the viscous term is insensitive to the strain rate level. With the parameter q_6=0.04 for Kaolin, shear strain rate ranging from $\dot{\gamma}^*$=10^{-5} to 10^{+7} corresponds to the magnitude of viscous function r_q ranging from 0.1 to 0.9.

Table 3. Model parameters for representing strain rate effects.

Symbol	Mechanism	Parameter designation
α_{c}	Volumetric	Secondary consolidation coefficient
$\dot{v}_0$	Volumetric	Initial volumetric creep strain rate
r_{TSR}	Shear	TESRA parameter representing decay rate $r_{\mathrm{TSR}} = r_1$
r_{Iso}	Shear	Isotach parameter representing max amplitude $r_{\mathrm{Iso}} = \alpha$
$r_{\dot{\gamma}}$	Shear	Isotach parameter for normalizing strain rate $r_{\dot{\gamma}} = 1/\dot{\varepsilon}_{\mathrm{r}}^{\mathrm{ir}}$
q_6	Shear	Isotach parameter for representing strain rate dependency $q_6 = m$

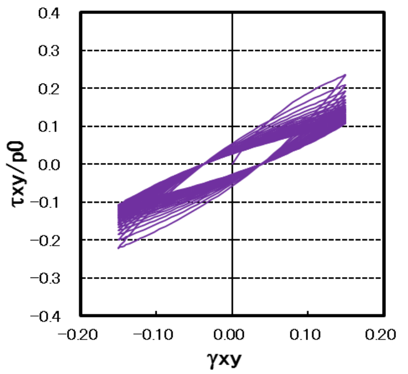
Model Performance Under Undrained Cyclic Loading

Performance of the proposed model is studied for its capability to simulate strain rate effects under undrained cyclic loading. Isotach parameters used for this study are r_{Iso}=0.5, $r_{\dot{\gamma}}$=10^6 s, and q_6=0.04 for representing Kaolin. Other parameters are the same as shown in Table 2 except for q_2 = 2, $r_{\varepsilon_{\mathrm{d}}^{\mathrm{c}}}$ = 0.085. Volumetric creep effect is not included in this analysis.

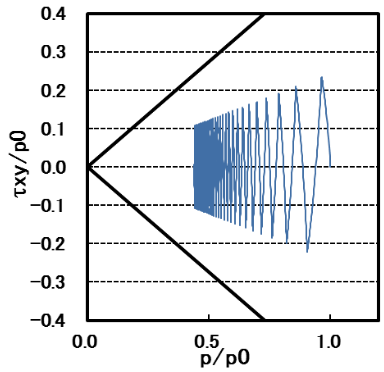
A clay element is initially consolidated at isotropic stress, then strain controlled tests are performed with a sinusoidal motion of 1Hz with a shear strain amplitude of 0.15, which corresponds to the peak shear strain rate of 15%/s. Computed results by the proposed model are shown in Figure 5 with reference to the computed results by non-viscous model. The results indicate that the proposed model is capable to simulate the strain rate effects of clay.

Conclusions

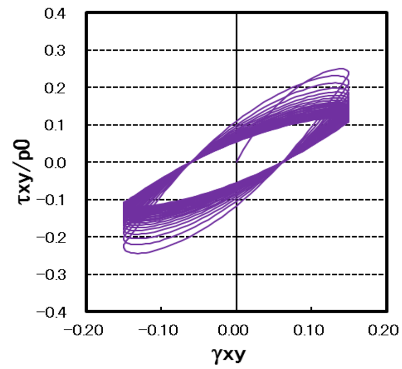
The proposed model based on the framework of strain space multiple mechanism model demonstrates the ability to overcome the limitations in the conventional elasto-plastic (or visco-elasto-plastic) models for clay. The proposed model is able to analyze arbitrary initial K_0 state which has significant effects on deformation due to seismic load and also to represent viscous effects of clay for earthquake loading which may include extremely high strain rate in shear strain. The application of the proposed model is expected to open the door to solve challenging problems of combined geotechnical hazards.



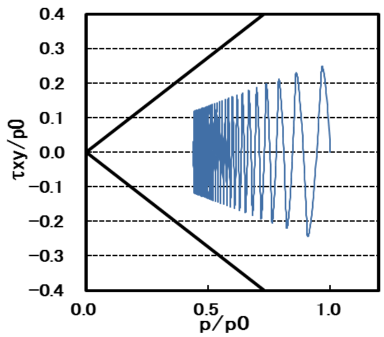
(a) Non-viscous hysteresis curve



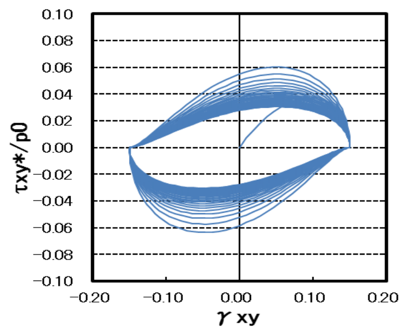
(b) Non-viscous stress path



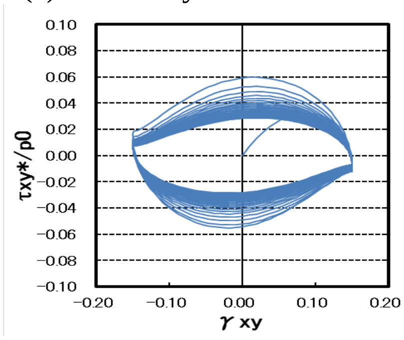
(c) Isotach hysteresis curve



(d) Isotach stress path



(e) Isotach viscous term



(f) TESRA viscous term

Figure 5 Performance under undrained cyclic loading

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