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Evaluation of 1-D Non-linear Site Response Analysis using a General Quadratic/Hyperbolic Strength-Controlled Constitutive Model

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ABSTRACT

Reliable estimates of 1-D non-linear seismic site response due to strong ground shaking or in soft soils requires appropriate representation of soil strength in addition to small-strain nonlinearity and cyclic response. The focus on representing small-strain soil behavior resulted in the development of several modified hyperbolic models to define the monotonic stress-strain (i.e. backbone) curve coupled with unloading-reloading (i.e. damping) behavior. Though such models can accurately capture small-strain behavior, in some cases the shear strength of soil is underestimated while in others it is overestimated leading to inaccurate estimates of site response. The authors introduce a General Quadratic/Hyperbolic (GQ/H) model which allows the shear strength of soil at failure to be defined while still providing the ability to represent small-strain stiffness nonlinearity. The GQ/H model is verified in comparison with the existing model, and the effect of properly modeling soil shear strength is demonstrated through application to total-stress cases studies.

Introduction

Characterizing the non-linear cyclic response of soils under dynamic loading requires consideration of the initial stress-strain curve, the unloading-reloading behavior, and the generation of excess pore pressure. Much of the work over the past 50 years has focused on the development and refinement of hyperbolic models that define the monotonic stress-strain (i.e. backbone) curve coupled with unloading-reloading (i.e. damping) behavior. These models are then fit to reference curves of normalized shear modulus and damping values as functions of shear strain. While such models can adequately characterize the small-strain behavior, the large-strain shear strength is typically left uncontrolled, allowing for unrealistic shear stresses to be developed with increasing shear strains. Strength corrections have typically been made manually (Hashash et al., 2010, Chiu et. al, 2008) because the shear strength of soil is underestimated in some situations while overestimated in others. Such corrections are often time-consuming and are highly subjective. Yee et al (2013) proposed the use of composite hyperbolic curves to control for the shear strength. This paper introduces a General Quadratic/Hyperbolic (GQ/H) model which allows the shear strength at failure to be defined while still providing the flexibility to represent the small-strain soil behavior. The unload-reload stiffness uses a non-Masing criteria via inclusion of a damping reduction factor introduced in an earlier model (MRDF model in DEEPSOIL) to match laboratory-measured damping curves. The GQ/H model is implemented in the site response software DEEPSOIL, and comparative analyses have been made with the commonly-used Modified Kondner-Zelasko model (Matasovic, 1993).

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General Quadratic Formulation

To capture the small-strain and large-strain behavior of a material, both the initial shear stiffness (G_{max}) and the shear strength at failure (τ_{max}) must be known. These values represent linear boundaries of stress-strain behavior in stress-strain space. A quadratic model can be used to join the two lines into a continuous curve because these linear boundaries are known to intersect at some reference shear strain. The proposed backbone curve has the form:

$$\frac{\tau}{\tau_{max}} = \frac{2\left({}^{\gamma}/_{\gamma_r}\right)}{1 + \left({}^{\gamma}/_{\gamma_r}\right) + \sqrt{\left\{1 + \left({}^{\gamma}/_{\gamma_r}\right)\right\}^2 - 4\theta_\tau\left({}^{\gamma}/_{\gamma_r}\right)}} \tag{1}$$

where τ is shear stress, τ_{max} is the shear strength at failure, γ is shear strain, γ_r is the reference shear strain, and θ_τ is a curve-fitting parameter. In this model, the reference shear strain maintains the original definition proposed by Kondner (1963) such that $\gamma_r = \tau_{max}/G_{max}$. The model is derived from a general quadratic equation that is simplified to a general hyperbolic equation. Hence the backbone curve is a general quadratic/hyperbolic form, and is labeled GQ/H model.

From consideration of several laboratory-obtained normalized shear modulus reduction curves, the proposed relationship for θ_τ has the form:

$$\theta_\tau = \theta_1 + \frac{\theta_2 \cdot \left(\frac{\gamma}{\gamma_r}\right)}{\theta_3 + \left(\frac{\gamma}{\gamma_r}\right)} \leq 1 \tag{2}$$

where θ_1, θ_2, and θ_3 are curve-fitting constants chosen to provide the best fit to the normalized shear modulus versus shear strain curves over a defined strain range. θ_1 and θ_2 must be selected such that $\theta_1 + \theta_2 \leq 1$.

Most commonly used nonlinear time-domain site response analysis codes use the extended unload–reload Masing rules to model hysteretic behavior. However, the hysteretic damping behavior calculated using the unloading-reloading stress-strain loops based on the Masing rules is known to overestimate the damping at large strains. The MRDF model (Phillips and Hashash, 2009) has been applied to the unloading-reloading relationship to provide better agreement with laboratory-measured damping curves. The model can also be used with porewater pressure generation models. The derivation and theoretical background of the GQ/H model are discussed in Groholski et. al (in preparation). The GQ/H model is implemented in DEEPSOIL. The performance of the model at the element level and in site response analysis is described in the following sections.

Calibration of Model Parameters

In the absence of site-specific data, empirical modulus reduction and damping curves such as EPRI (1993), Vucetic and Dobry (1991), and Darendeli (2001) are commonly used in site response analysis to represent dynamic soil behavior. Curves proposed by Darendeli (2001) have been used to demonstrate the application of the GQ/H model to site response analyses.

The fitted curves are compared to fits obtained using the Modified Kondner-Zelasko model (Matasovic, 1993), which is one of the most commonly used backbone formulations in 1-D site response analyses. Figure 1 shows the GQ/H model fitting procedure for a modulus reduction curve for a vertical effective stress of 169.9 kPa, PI of 25, OCR of 1, and shear strength of 40 kPa, which will be used for the station Apeel #2 (A02) in Redwood City, California site response analysis (described in the next section). The reference curve was obtained from Darendeli (2001) equations.

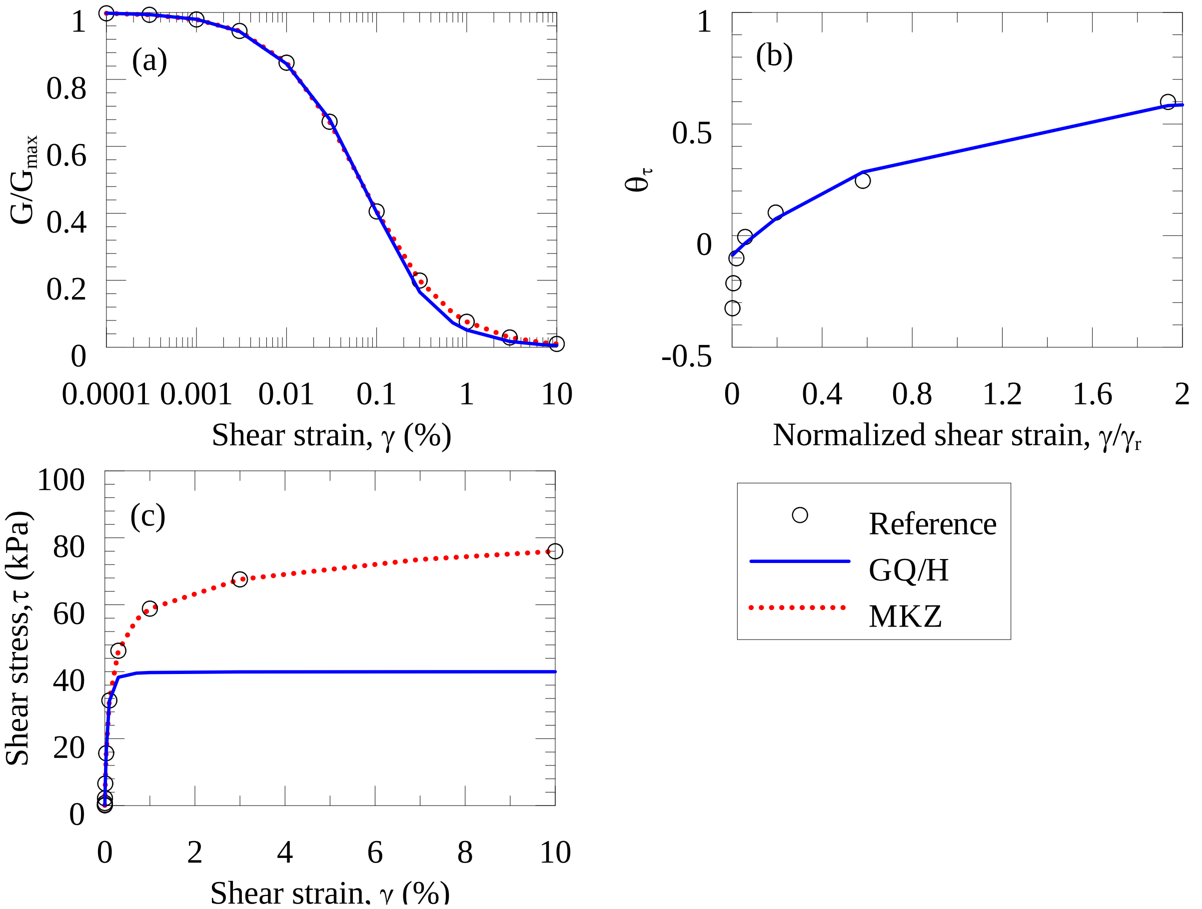

Figure 1 GQ/H model fitting procedure for clays with a vertical effective stress of 169.9 kPa, PI of 25, OCR of 1, and shear strength of 40 kPa (reference curve from Darendeli (2001)): (a) normalized shear modulus reduction; (b) θ_τ parameter relationship to normalized shear stress; and (c) shear stress-shear strain response.

The target shear stress (τ_{max}) is estimated to be 40 kPa. The parameter θ_τ is fitted by the hyperbolic relationship (Eq. 2) with coefficients θ_1, θ_2, and θ_3 of -0.09, 1.02, and 1, respectively. The MKZ and GQ/H models both match the reference G/G_{max} curve (Figure 1a). The MKZ model does not have the capability to match the target shear stress at large shear strains. However, the GQ/H model yields shear stress that approaches the target shear strength at large shear strains (Figure 1b).

Comparative Total-Stress Site Response Analyses

The performance of the GQ/H model in comparison with the MKZ model is demonstrated at two sites whose profiles correspond to Apeel #2 (A02) in Redwood City, California; and Service Hall array (SHA) at the Kashiwazaki-Kariwa Nuclear Power Plant (KKNPP) in Japan. Figure 2 (a) and (b) show geology and shear wave velocity profiles, respectively, for station A02 (Baturay and Stewart 2003, http://www.cee.ucla.edu/faculty/stewart/research). The site consists of predominantly soft clay overlying stiff clays and shale.

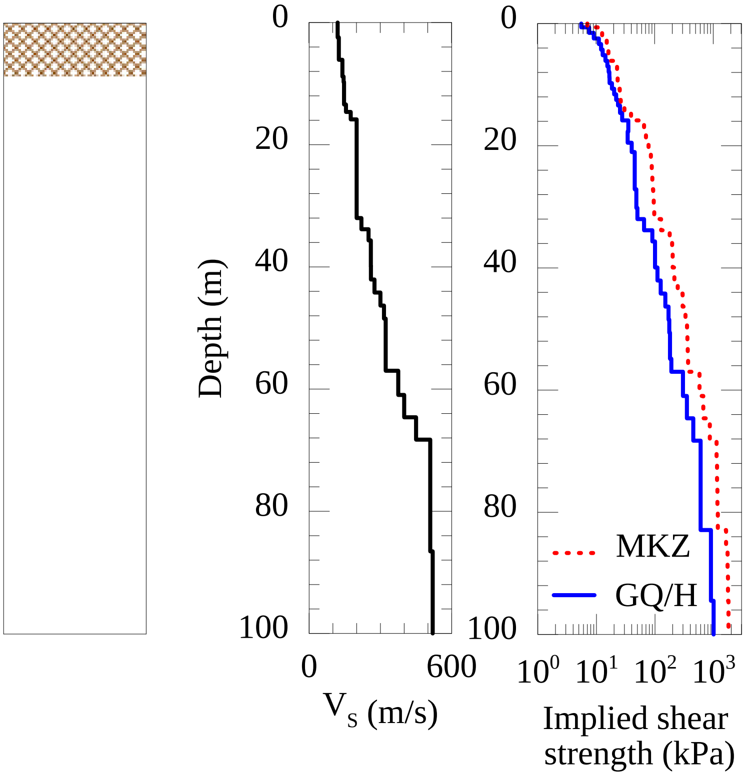

Figure 2 Profiles of (a) Geology and (b) shear wave velocity (V_S) of station Apeel #2 (A02) in California, and (c) implied shear strength for both MKZ and GQ/H models.

The assigned modulus reduction and damping curves for the soil column are derived from Darendeli (2001) relationships. Figure 3 shows normalized shear modulus, damping ratio, and shear stress -shear strain curves at three selected depths. The shear strengths are for the profile is defined as $0.22\sigma_v'$ above 16 m and below 16 m ranges from $0.3\sigma_v'$ to $1.0\sigma_v'$ increasing with depth. The use of the MKZ model fit of the modulus reduction curves results in significant over estimation of the shear strength. The GQ/H model adequately captures the target shear strength as this is a direct model input. No manual adjustment is required to achieve the shear strength profile as has been done so far when these types of conditions are encountered.

Figure 4 shows the results of site response analyses subjected to the strong ground motion (Record Sequence Number (RSN): 4876) obtained from the NGA-West2 database (Ancheta et al. 2014). For each of the MKZ and GQ/H models, equivalent linear (EL) and nonlinear (NL) site response analyses were conducted. Peak ground acceleration and peak shear strain versus depth are plotted. The computed surface response spectra as well as the input motion response spectrum are also plotted.

For both the EL and NL analyses the surface response computed with the GQ/H model is lower than that using the MKZ model especially around a period of 1 Hz and at high frequencies. As the GQ/H model has lower strengths than the MKZ model, larger strains are computed for the GQ/H model as would be expected.

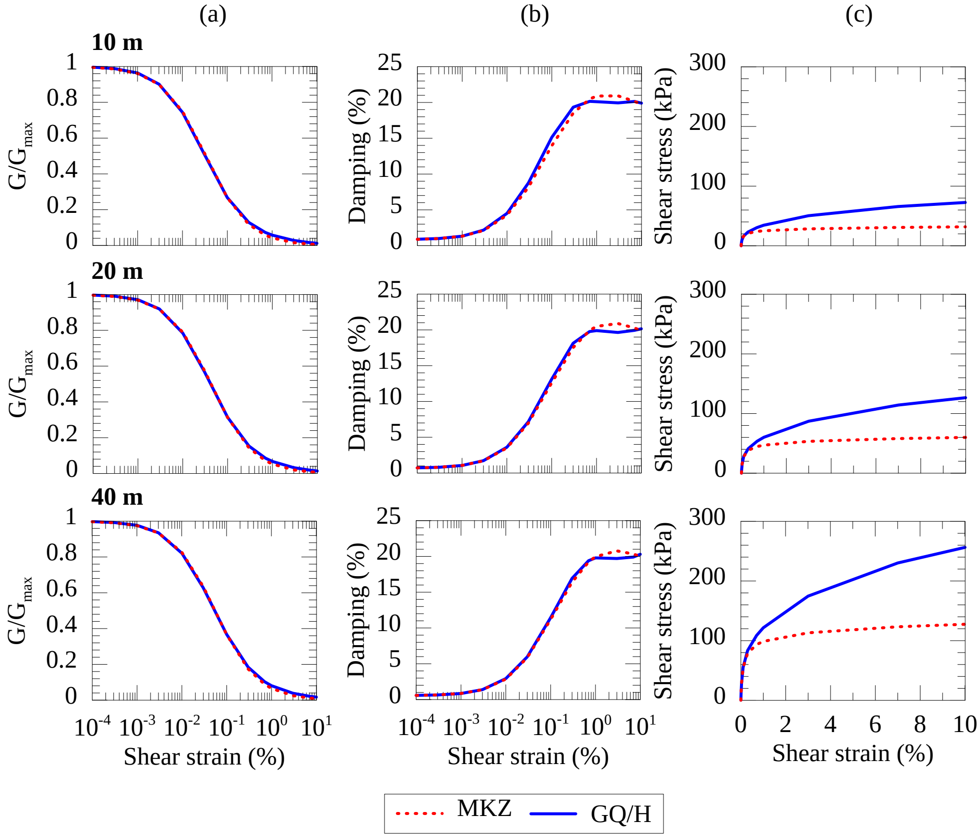

Figure 3 Normalized shear modulus (a), damping ratio (b), and shear stress (c) curves for station Apeel #2 (A02), California at selected depths (10, 20, and 40 m) for both MKZ and GQ/H models.

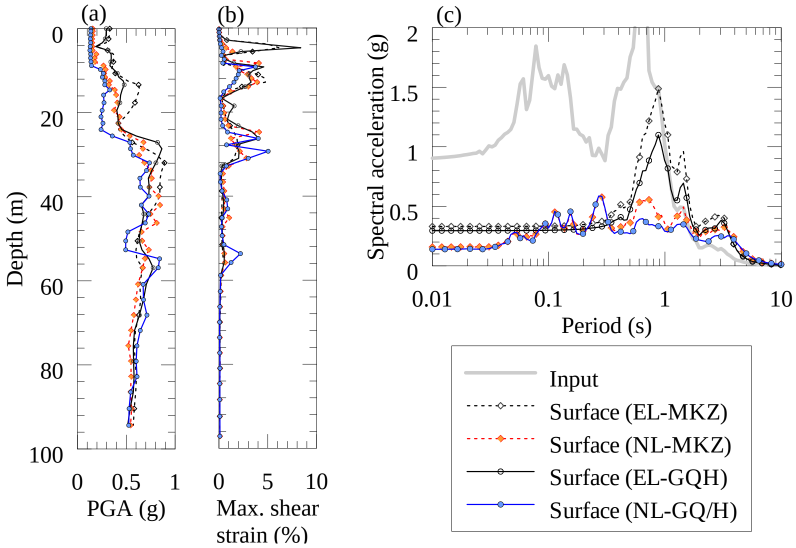

Figure 4 Results of site response analysis for station Apeel #2 (A02) subject to the NGA-West 2 strong ground motion measure (RSN: 4876): (a) PGA profiles; (b) maximum shear strain profiles; and (c) surface response spectra.

Figure 5 (a) and (b) show geology and shear wave velocity profiles, respectively, for station Service Hall array (SHA) at the Kashiwazaki-Kariwa Nuclear Power Plant (KKNPP) in Japan (Yee et al. 2013). The implied shear strengths and friction angle of sand layers are corrected using the GQ/H model based on the friction angle measured by Yee et al. (2013) using consolidated-drained triaxial compression tests as shown in Figure 5 (c and d). Unlike the case of station A02, the strengths corrected by the GQ/H model are greater than those estimated by the MKZ model. Figure 6 shows the results of site response analyses for station SHA subject to the strong ground motion (RSN: 143) obtained from the NGA-West2 database (Ancheta et al. 2014). The maximum shear strains for nonlinear and equivalent-linear analyses using the MKZ model are approximately 9 % and 7 %, respectively, at depths less than 24 m. When the shear strengths are corrected using the GQ/H model, the maximum shear strains decrease to less than 2 % for both nonlinear and equivalent-linear analyses, respectively. There is also significant reduction in shear strains at depths between 50 m and 56 m. This reduction in shear strains is due to the increased shear strengths. As a result of the decreased maximum shear strain, the PGA values using the GQ/H model generally increased at depths less than 16 m compared to those using the MKZ model. The response spectra on the ground surface are shown in Figure 6 (c). The spectral accelerations for a nonlinear analysis using the GQ/H model are greater than those using the MKZ model at periods less than 5 s. The spectral accelerations for an equivalent-linear analysis using the GQ/H model are greater than those using the MKZ model at periods less than 2 s.

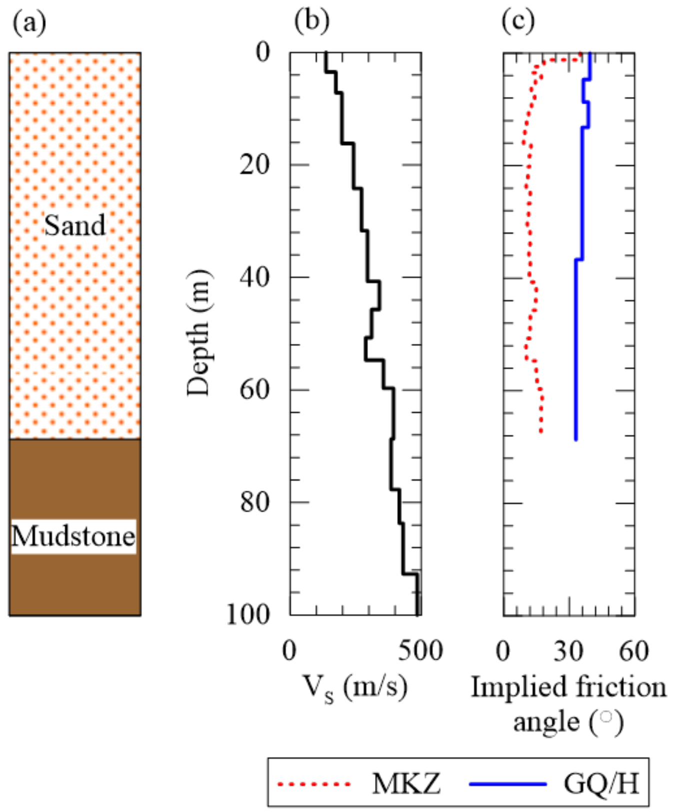

Figure 5 Profiles of (a) Geology and (b) shear wave velocity (V_S) of station Service Hall array (SHA) at the Kashiwazaki-Kariwa Nuclear Power Plant (KKNPP) in Japan, and (c) implied friction angle for both MKZ and GQ/H models.

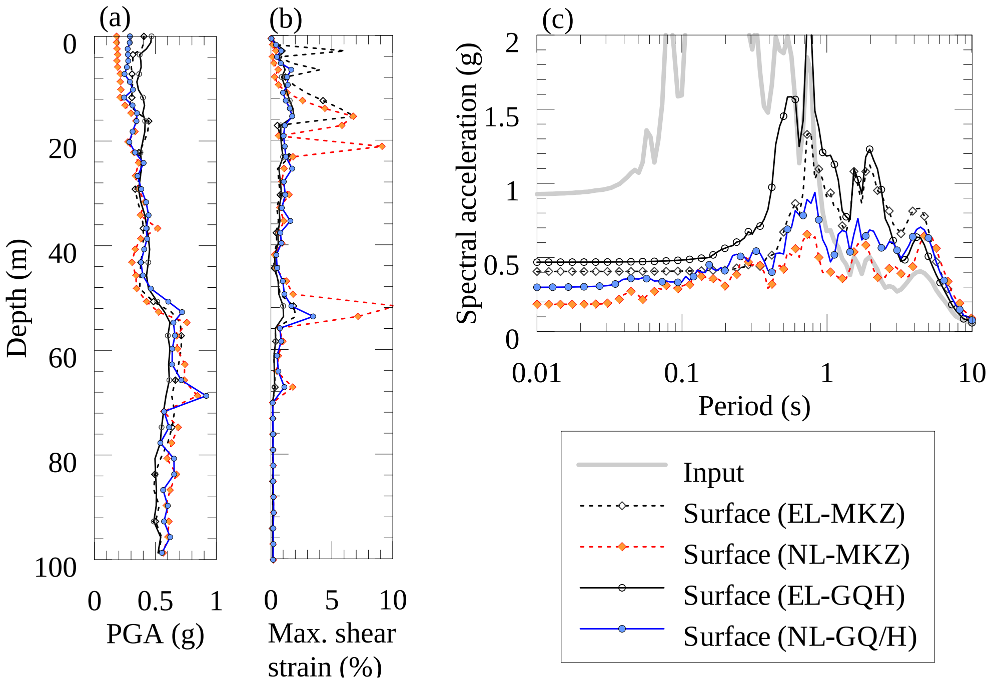

Figure 6 Results of site response analysis for the Service Hall array (SHA) at the Kashiwazaki-Kariwa Nuclear Power Plant (KKNPP) subject to the NGA-West 2 strong

ground motion measure (RSN: 143): (a) PGA profiles; (b) maximum shear strain profiles; and (c) response spectra on the ground surface.

Conclusion

This paper introduces a new generalized quadratic/hyperbolic (GQ/H) model that can be used to represent both small strain nonlinear behavior and shear strength of the soil. The model is a simplified one dimensional shear stress-shear strain model that overcomes limitations of an available model widely used in nonlinear site response analysis. Site response analyses using the proposed model demonstrate that computed site response is quite sensitive to the implied shear strength in the soil model especially for soft soil sites. Validation of the proposed model using field measurements of soil shear strengths and ground motion recorded from vertical arrays is desirable in the future.

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