Identification of Incident and Site-To-Site Factors for Ground Motion Coherence Based On Very Dense Seismic Array Observations

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ABSTRACT

The spatial variation models have been proposed in past researches, whereas the models cannot include the difference of site condition. In order to account for the effect, we suppose that ground transfer function is a stochastic variable, and it enables us to introduce a difference of site conditions into the coherence. We assume the site-to-site factor independent of the earthquake events, and the incident factor independent of site pairs. Both the site-to-site and incident factors are identified from the observed coherence. We adopt the method to the array data observed in Furukawa district, Japan. The smaller site-to-site factor is well correlated to the region where the basement depth steeply changes. This suggests that the coherence on the complex subsurface structure may be modeled by a combination of incident factor depending on the earthquake events, and the site-to-site factor reflecting the detail 3-D subsurface model.

Introduction

Seismic ground motions spatially vary depending on the spatial intervals and also the relative site conditions. The spatial variation greatly influences the response of long linear structures and networks of infrastructure systems.

In early studies, dense seismic arrays were installed on a flat plain, and the variation has been discussed based on the observed seismograms. For example, SMART1 array (Bolt et al., 1982) was operated at the northeastern of Taiwan during 1980-1990, and the smaller scale array, LSST array, was installed within the southwest quadrant of the SMART1 array. The observed data exhibited the loss of correlation between the seismograms as increasing the spatial intervals between the seismic stations and being higher frequencies. The physical reasons have been explained by wave passage effect, extended source effect, scattering effect, attenuation effect, etc. (e.g., Zerva, 2009), and several models have been established by reflecting the physical insights. The most of models, however, assume homogeneous decay of correlation by taking the spatial average of the data set, and several researches have discussed the effect of site condition differences on a case-by-case (e.g., Liao et al., 2007).

On the other hand, variation of site condition is essential to discuss the micro zonation of seismic hazard at a particular area. The site condition, e.g., soft deposit soil or hard rock, is classified based on the several geophysical surveys, and it is adopted to quantify the local site

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amplification, such as Vs30 (e.g., Borcherdt, 1994). However, the loss of spatial correlation has not been well considered in constructing both the site condition and the site amplification because they are not assumed to be homogeneous patterns. Therefore, it is necessary to establish the theory to combine the spatial correlation model and the relative site conditions.

In the modeling, we focus on Furukawa array, which was installed in the downtown of Furukawa district, Osaki city, Miyagi after the 2011 Off the Pacific Coast of Tohoku Earthquake, Japan (Goto et al., 2012). They are motivated to clarify the reason why the ground motion damages were concentrated at a particular area in the downtown, where is the scale of about 0.5x1.0km². In 2014, 36 seismic stations have been installed in 2x3km² area. From the observations, it is revealed that the site amplification is not spatially uniform due to the difference of ground subsurface structures (Inatani et al., 2013). The Furukawa array is suitable observation to discuss the spatial correlation influenced by variable site conditions.

In this study, we propose a method to extract the contribution from the site condition from the observed coherency, which is a typical variable on spatial correlation. The factor related to the homogeneous decay of the coherency is named incident factor, and one related to the site condition is named site-to-site factor. Both the factors are estimated from the data set in Furukawa array, and discuss the relation to the ground subsurface structures in the area.

**Coherency of Ground Motion**

Coherency is one of the major variables on the spatial correlation of seismic ground motion, and it measures the similarity of two ground motion records. The coherency (lagged coherency) between the waveforms observed at site $i$ and site $j$ is defined by the following equation.

$$ |\gamma_{ij}(\omega)| = \frac{|S_{ij}(\omega)|}{\sqrt{S_{ii}(\omega)} \sqrt{S_{jj}(\omega)}} $$

where $S_{ij}(\omega)$ is cross spectra and $S_{ii}(\omega)$, $S_{jj}(\omega)$ are power spectra at site $i$ and $j$, respectively. $\omega$ is angular frequency. When two waveforms are exactly the same, the value of the coherency is equal to 1. When two waveforms are not similar, it is less than 1. Empirically, the coherency depends on the frequency and spatial site intervals. For lower frequency range and shorter intervals, the value of the coherency is close to 1. For higher frequency range and longer intervals, it decreases.

Figures 1 show the coherency calculated from the observed waveforms at F03 and F09 stations in Furukawa array for the event occurred on August 4, 2013. Cross and power spectra are estimated by smoothed in frequency range with 0.4Hz Parzen window, and the coherency is calculated from Equation 1. The spatial interval between F03 and F09 stations is 256m. The coherencies have large values in about 0.4-0.6Hz, and gradually decrease in the higher frequencies. Figure 2 shows the coherencies at 0.4Hz calculated from every pair of the stations for the same event data. The spatial intervals are distributed in the range of 60-2000m. The coherency decreases in the longer distances. As shown in Figs.1-2, the coherence becomes independent of frequency and distance in high frequency and longer distances. We assume it the residual values of coherence in higher frequency and longer distances because of observation
noise. We estimate the values about less than 0.33, which is consistent with the past researches (Abrahamson et al., 1991), and it is not adopted to the data analysis.

Figure 1. Frequency contents of coherency between F03 and F09 stations in Furukawa array for event August 4, 2013.

Figure 2. Spatial decay of coherency in Furukawa array for event August 4, 2013.

Figure 3. The coherency distribution between F14 stations and the others for events on August 30, 2012 (top panels) and December 7, 2012 (bottom panels). Thin and thick lines draw concentric circles from F14 station with 100m and 500m intervals, respectively.
Figures 3 shows the maps of coherency between the center site (F14 station) and the others for the event on August 30, 2012 and December 7, 2012. The thin and thick lines are the concentric circles from F14 station with the 100m and 500m intervals, respectively. If the coherency is homogeneous decay in the distance, the coherency in maps independent of the direction from the center site. However, the maps indicate that the coherency in the southeast direction is larger than in the northwest direction, and it varies with the site even though the sites are almost the same distance. Figure 4 shows the depth to the support basement modeled by Inatani et al. (2013). The depth at F14 stations (marked by white triangle) is similar to one at the stations located in southeast direction. This implies that the coherency may have some tendencies independent of the earthquake events, and may be associated with the distribution of ground subsurface structure.

![Map of coherency](image)

**Figure 4. Depth distribution of the support basement in Furukawa array (Inatani et al., 2013)**

### Site-to-Site Factor on Coherency

Ground motion on the free surface $Y(\omega)$ is represented by incident ground motion $X(\omega)$ and ground transfer function $H(\omega)$.

$$Y(\omega) = H(\omega)X(\omega).$$  

(2)

The cross spectra between site $i$ and $j$ are evaluated from the expected values of the cross terms of $Y(\omega)$, $S_{Yij}=E[Y_i^*Y_j]$, and the power spectra are also evaluate from the expected values of the power, $S_{Yi}=E[Y_i^2]$. Then, the coherency on the free surface between site $i$ and $j$ is calculated, as follows:

$$\gamma_{Yij}(\omega) = \frac{E[Y_i^*Y_j]}{\sqrt{E[Y_i^2]}\sqrt{E[Y_j^2]}} = \frac{E[H_i^*H_j]}{\sqrt{E[H_i^2]}\sqrt{E[H_j^2]}} \gamma_{Xij}(\omega).$$  

(3)
where $H_i$ and $H_j$ are the transfer function at site $i$ and $j$, respectively. Equation 3 assumes the independence of probability between the transfer function and the incident ground motion.

If the transfer functions are deterministic variables, the expected values are constant. Then,

$$|\gamma_{ij}(\omega)| = |\gamma_{X_{ij}}(\omega)|.$$  \hspace{1cm} (4)

Equation 4 means that the effect of ground amplification is not included in the observed coherency on the free surface even though the transfer functions do not coincide. It implies that the observed coherence is not affected from the difference of site conditions. However, the actual observed coherence has some relations to the subsurface structure in the area discussed in Figs.3-4.

Therefore, we assume the ground transfer functions to be stochastic variables, and then a factor $C_{ij}(\omega)$ reflecting the difference of transfer functions remains.

$$|\gamma_{ij}(\omega)| = |C_{ij}(\omega)||\gamma_{X_{ij}}(\omega)|,$$

$$|C_{ij}(\omega)| = \frac{E[H_i^*H_j]}{\sqrt{E[|H_i|^2]E[|H_j|^2]}}.$$ \hspace{1cm} (5)

The factor $C_{ij}(\omega)$ is named site-to-site factor between site $i$ and $j$. It enables us to include the frequency contents of the ground transfer functions. The stochastic property in the transfer function is caused by the amplification variations in the events, which are well observed in the spectral ratios (e.g., Goto et al., 2012, Goto et al., 2013). One of the major reasons to exhibit the amplification variations may be the variation of incident angles depending on the earthquakes. In order to explain the irregular distribution of the coherency (Fig.3), we accept the ground transfer functions as stochastic variables, and identify the site-to-site factors in Furukawa array.

**Identification of Incident and Site-to-Site Factors**

In order to identify the site-to-site factors from the observed coherency on the free surface, we adopt the following two assumptions.

- Incident factor varies with the earthquake events, while it has the same stochastic properties at all sites during the specific earthquake. In other words, the incident factors are homogeneous.
- Site-to-site factor varies with the pair of the sites, while they have the same stochastic properties over the earthquake events.

In addition, we introduce the homogeneous coherency models for the incident factor as the function of the separation distance $r_{ij}$ by Luco and Wong (1986), and then the coherency model is represented by following equation.

$$|\gamma_{Y_{ij}}(\omega)| = |C_{ij}(\omega)| e^{\alpha r_{ij}}.$$ \hspace{1cm} (7)
where left hand side of the above equation is the observed coherency between site \(i\) and \(j\) during the event \(e\), \(|C_{ij}(\omega)|\) is the site-to-site factor and \(\alpha'(\omega)\) is the negative parameter for the incident factor.

We apply two-stage regression; the incident factors are identified in the first stage, and the site-to-site factors in the second stage. In order to avoid the unreliable data, we select the coherency data under satisfying the three conditions; 1) the coherency is larger than 0.33, 2) the site-to-site distance is shorter than 500m, and 3) the station pair observes more than three events. The data sets for four major events (March 27, 2012, August 30, 2012, December 7, 2012, and August 4, 2013) are selected by taking account for the S/N ratio. Figures 5 shows the comparison between the observed coherencies and the incident factors for the events on August 30, 2012 and December 7, 2012. The incident factors pass through the center of the observed data. The values of \(\alpha'(\omega)\) are different depending on events.

The site-to-site factors are estimated from an average of the residuals between the coherency and the incident factors over the earthquake events. Figure 6 shows distributions of the identified site-to-site factors. The red bars indicate pairs of the larger site-to-site factors than 1.0, and the blue bars indicate ones of the smaller than 1.0. We compared the distribution of site-to-site factors with the distribution of the support base ment (Figure 4). The vicinity of the sites connected by blue bars agrees well with the sites where the depth to basement steeply changes. The vicinity of the sites connected by red bars agrees with the sites where the depth to basement is almost flat.

The numerator of \(|C_{ij}|\) (Equation 6) contains the covariance of transfer functions between sites, and the denominator contains each variance. When the value of \(|C_{ij}|\) is large, the covariance of transfer functions is large and/or each variance is small. If the steep change of the basement causes the large amplification variations, the denominator of \(|C_{ij}|\) becomes small. It implies that the small site-to-site factor may be related to the appearance of steep change of basement.

![Figure 5. Comparison between observed coherency and the incident factors for events on August 30, 2012 and December 7, 2012 (0.4Hz; UD components).](image-url)
Conclusions

In this study, we propose a method to extract the contribution from the site condition from the observed coherency. Incident and site-to-site factors are identified from the observed coherency in Furukawa array, and the smaller site-to-site factors are well correlated with the region where the basement steeply changes. The site-to-site factor theoretically appears when the transfer function is a stochastic variable, and it may reflect the irregularity of the subsurface structure.

In order to discuss how to model the site-to-site factors in a particular region, we quantitatively construct the relation between the irregularity of subsurface structure and the transfer function variations.

References


