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Analysis of Foundation Damping from Theoretical Models and Forced Vibration Testing

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ABSTRACT

Foundation damping incorporates the combined effects of energy loss due to waves propagating away from the vibrating foundation (radiation damping), as well as hysteretic action in the soil (material damping). Two closed-form solutions for foundation damping of a flexible-based single degree-of-freedom oscillator were derived from first principles where the inherent (structural) damping ratio, radiation damping and the soil hysteretic damping ratio appear as variables. Since both formulations are theoretically defensible, yet provide numerically distinct results, we look to case histories for validation. In addition to previously established case histories we evaluated data derived from forced vibration testing of a field test structure installed at two sites in California. Parametric system identification was employed to evaluate the fixed-base and flexible-base first mode period and damping ratio during tests performed across a wide frequency range and multiple load amplitudes. Foundation damping derived from these results is shown to favor one of the theoretical models over the other.

Introduction

Following the early work by Parmelee (1967), foundation damping as a distinct component of the damping of a structural system was introduced as part of Bielak's (1971) derivation of the replacement (flexible-base) single-degree-of-freedom (SDOF) system and was later refined by Veletsos and Nair (1975), Roesset (1980) and others. The work was predicated on the need to evaluate the effects of soil-structure interaction (SSI) on the seismic response of nuclear power plants. Based on that need, alternative sets of equations were developed to predict foundation damping of a rigid circular foundation resting on a uniform elastic halfspace. Due in part to the convenience of its application in the specification of seismic demands using response spectrum (force-based) or pushover (displacement-based) methods of analysis, foundation damping has more recently appeared in several seismic design guidelines for building structures (e.g., ASCE, 2006, 2010).

We summarize results of alternative derivations of foundation damping, originally presented by Givens et al. (2015), which match the response of a single-degree-of-freedom (SDOF) equivalent fixed-base oscillator to that of the flexible-base oscillator. The results show some differences from classical solutions. We describe field-scale forced vibration testing that was performed in part to measure foundation damping. We briefly review system identification processes and the necessary input-output pairs required to evaluate first-mode period and

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damping for the fixed-base and flexible-base conditions. Foundation damping is derived from the results and compared to the theoretical derivations.

Foundation Damping Solutions

The replacement SDOF approach has led to the overall damping of soil-foundation-structure systems (β_o) being partitioned into components associated with soil-foundation interaction (including hysteretic and radiation damping, referenced as "*foundation damping*"), β_f, and damping within the structural system, β_i, (referenced as "*structural or inherent damping*") as follows:

$$\beta_o = \beta_f + \frac{1}{(\tilde{T}/T)^n}\beta_i \tag{1}$$

Equation 1 appears in many seismic design guidelines with n = 3. Givens et al. (2015) show that the exponent n depends on the type of damping (i.e., ideally viscous or general, including hysteretic). Historical precedent and system identification studies often dictate values of β_i in the range of 2-5% in structural design practice for buildings responding in the elastic range (FEMA 440, 2005). The most challenging aspect of deriving β_o is the evaluation of β_f since β_i can be established from the aforementioned guidelines and period lengthening $(\tilde{T}/T)$ analysis is relatively straightforward.

We consider an SDOF oscillator of height h and stiffness k supported on a flexible-base condition consisting of a horizontal spring (k_x) and rotational spring (k_{yy}). The total deflection $(\tilde{\Delta})$ of the oscillator mass (m) from the application of a horizontal force (F) results from horizontal deflection of the structural spring and rigid body displacement of the structural mass from horizontal foundation displacement (u_f) and base rotation (θ).

$$\begin{aligned}\tilde{\Delta} &= \frac{F}{k} + u_f + \theta \cdot h \\ \tilde{\Delta} &= \frac{F}{k} + \frac{F}{k_x} + \left(\frac{F \cdot h}{k_{yy}}\right)h\end{aligned} \tag{2}$$

Considering that the replacement oscillator has deflection $\tilde{\Delta}$ in response to force F, the stiffness of the replacement oscillator $\tilde{k}$ can be taken as the ratio of $\tilde{\Delta}$ to F.

The formulation of the damping component of equivalent oscillator requires consideration of the imaginary parts of stiffness terms. Damping results from the phase difference between the real and imaginary parts of the oscillator response. Givens et al. (2015) provide two derivations for foundation damping. Both begin by dividing the right and left sides of Equation 2 by force F, recognizing that $\tilde{\Delta}/F$ is equivalent to an effective flexible stiffness of the replacement oscillator $\tilde{k}$, and generalizing each term for dynamic loading through the introduction of complex-valued stiffness terms (indicated by an overbar), as follows:

$$\frac{1}{\bar{\tilde{k}}} = \frac{1}{\bar{k}} + \frac{1}{\bar{k}_x} + \frac{h^2}{\bar{k}_{yy}} \tag{3}$$

There are two ways to proceed from Equation 3. The first approach is similar in some respects to prior work by Bielak (1971), Aviles and Perez-Rocha (1996) and Maravas et al. (2014). We separate Eq. 3 into its real and complex parts, and upon dropping higher-order (β^2) terms, operate exclusively on the imaginary part to evaluate the effective damping of the replacement oscillator. The foundation damping is then readily derived from the system damping. As shown by Givens et al. (2015), this leads to the following expression for β_f:

$$\beta_f = \left[1 - \frac{1}{(\tilde{T}/T)^2}\right]\beta_s + \frac{1}{(\tilde{T}/T_x)^2}\beta_x + \frac{1}{(\tilde{T}/T_{yy})^2}\beta_{yy} \tag{4}$$

where β_s represents the soil (hysteretic) damping, β_x and β_{yy} represents the radiation damping from foundation displacement in the horizontal and rotational direction, respectively.

In the second approach, both the real and complex parts of Eq. 3 are retained in developing expressions for the dynamic properties of the replacement oscillator (i.e., the replacement oscillator response matches the flexible-base oscillator response in both amplitude and phase). The resulting expression for foundation damping is:

$$\beta_f = \frac{1}{\left(\tilde{T}/\left|\bar{T}_x\right|\right)^2}\left(\beta_x + \beta_s\right) + \frac{1}{\left(\tilde{T}/\left|\bar{T}_{yy}\right|\right)^2}\left(\beta_{yy} + \beta_s\right) \tag{5}$$

This approach is similar in some respects to that of Veletsos and Nair (1975). Both solutions are presented with fictitious vibration periods in the denominator, calculated as if the only source of the vibration was foundation translation or rotation, with the structure being rigid, as shown in Eq. 6 (Givens et al., 2015).

$$T_x = 2\pi\sqrt{\frac{m}{k_x}} \quad T_{yy} = 2\pi\sqrt{\frac{m}{k_{yy}}} \quad \bar{T}_x = 2\pi\sqrt{\frac{m}{\left|\bar{k}_x\right|}} \quad \bar{T}_{yy} = 2\pi\sqrt{\frac{m}{\left|\bar{k}_{yy}\right|}} \tag{6}$$

It should be noted that the absolute values in Eqs. 5 and 6 are to make the expression for β_f real-valued. Further explanation of this issue is provided in Givens et al. (2015).

In Figure 1, we plot foundation damping derived from the two solutions against the ratio $\left(h/(V_s T)\right)$, which is often called the wave parameter (Veletsos, 1977). The wave parameter can be thought of as a structure-to-soil stiffness ratio because (h/T) represents the stiffness of a structure's lateral force resisting system in velocity units whereas soil shear wave velocity V_s is related to the soil shear stiffness. In Figure 1, foundation damping solutions are given for circular foundations and various structure height aspect ratios (h/r). Translation modes of foundation vibration dominate for small height aspect ratios (< 1), and rocking dominates for larger height aspect ratios (> 2). Veletsos and Wei (1971) impedance functions were used for the two sets of solutions shown in Figure 1.

Both the translational and rotational terms are affected by the introduction of imaginary terms in Eq. 5 (relative to the terms in Eq. 4) by amounts ranging from 0-10% for the translational term and 0-16% for the rotational term. These differences have negligible influence at larger height aspect ratios (e.g., $h/r \geq 2$), but substantial influence as the height aspect ratio is reduced. In all cases, Eq. 4 results in more foundation damping than does Eq. 5.

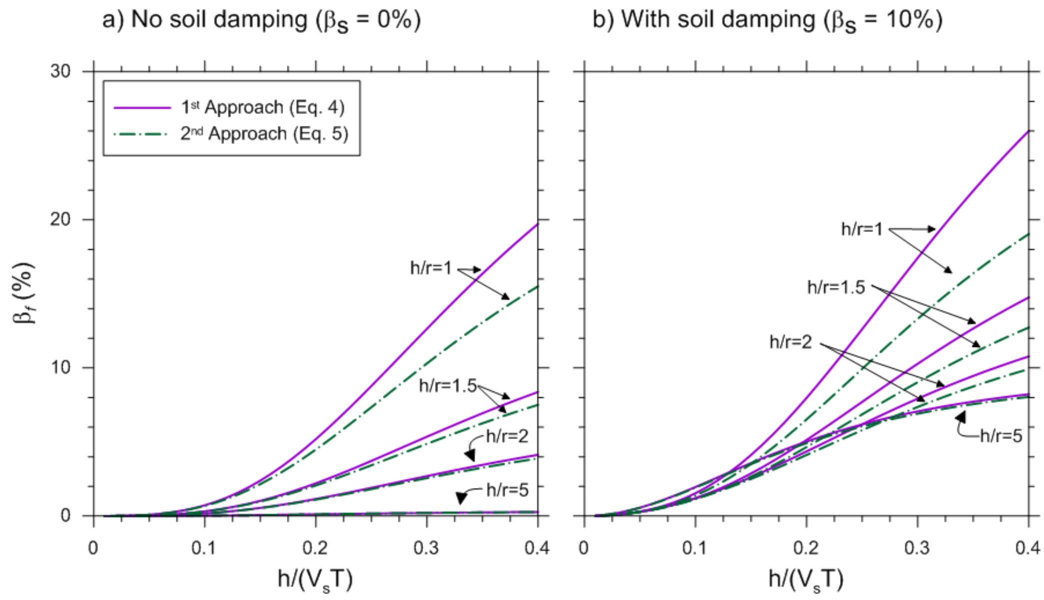

Figure 1. Comparison of foundation damping solutions (Equations 4 and 5) results for a rigid, massless, circular disc supported on a homogenous isotropic halfspace with ν = 0.33 and hysteretic soil damping a) β_s = 0% and b) β_s = 10%.

Field Testing

The importance of field testing to measure SSI effects is associated with inherent limits of foundation impedance models, such as those used in Eq. (2)-(5). Such models are typically developed for idealized conditions such as linear soil response, rigid foundations, and depth-invariant soil properties. Thus, testing is needed to evaluate model applicability for realistic field conditions and to guide the selection of input parameters. NIST (2012) summarizes existing models for predicting the stiffness and damping of foundation-soil interaction and current recommendations for adapting such models to field conditions. NIST (2012) also summarizes prior testing (laboratory and field) of soil-structure system responses. Field test results are limited; extending the available dataset therefore represents a substantial research need.

A sequence of unique experiments in which the same structure was subjected to forced vibrations at multiple sites representing varying degrees of base flexibility was performed by Star et al. (2015). The structure consists of a rectangular steel moment frame with removable cross-bracing and a reinforced concrete roof and foundation, as depicted in Figure 2 in its unbraced and braced configurations. An essentially fixed base condition was achieved in testing within a structural laboratory, whereas medium-stiff and soft soil conditions representing flexible base conditions are present at two field test sites. The shallow soils at the medium-stiff site consist predominantly of sand and silt with some clay with an average shear wave velocity of about 190 m/s. The shallow soils at the soft site consist predominantly of silt and clay with an average shear wave velocity of about 95 m/s. Forced vibrations were applied on the top slab and foundation mat with two shaker systems that impart small- and large-force demands. Specimen responses were recorded with accelerometers, pressure cells, and displacement transducers (as shown in Figure 2). A data acquisition system was used with precise time stamping, which is important for interpretation of damping effects. The test structures were also instrumented to record earthquakes for several months between tests and following the completion of testing. Details of the test structure configuration are given in Star et al. (2015).

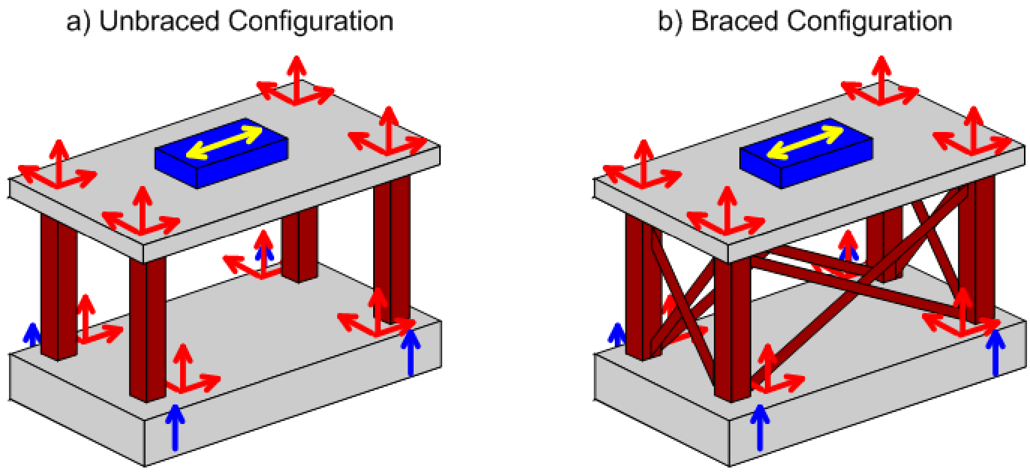

Figure 2. Field test structure depicted in an a) unbraced configuration and b) braced configuration. Arrows represent the generalized locations of the recorded accelerations (red), displacements (blue) and applied shaker force (yellow). Shaker force was also applied in the transverse direction and on the foundation mat. Pressures cells under mat not shown for clarity.

The full data set from testing at the laboratory and two field test sites can be found online at https://nees.org/warehouse/project/637. Each experiment included multiple trials with varying test conditions.

Evaluation of Field Testing Data

Foundation damping is not measured directly from field performance data; rather system identification analyses are performed to evaluate fixed- and flexible-base properties of structures, from which foundation damping is computed using Eq. 1.

We apply parametric system identification procedures in which the properties of a parametric model of the system are evaluated for a given input-output pair of motions (e.g., Stewart and Fenves, 1998). A crucial element of our system identification is the selection of appropriate input-output motions to evaluate fixed-base and flexible-base properties of the structure.

For the case of forced vibration testing, the input-output pairs for application to SDOF structural systems have been evaluated by Tileylioglu (2008) by solving equations of motion in the Laplace domain, with results in Table 1. Input-output pairs for more general application involving multi-degree of freedom structures have yet to be identified.

Parametric system identification evaluates a transfer function surface in the Laplace domain by fitting an underlying model (Stewart and Fenves, 1998). The locations of peaks in this surface can be related to modal periods and damping ratios. Figure 3 shows an example of transfer functions from parametric system identifications procedures using data from the aforementioned forced vibration tests where a large-force shaker was used. The peaks occur at modal frequencies, with the first distinct peak representing first-mode response. Also shown in Figure 3 are transmissibility functions from non-parametric system identification procedures, which represent the ratio of output/input signals without fitting an underlying model. The peaks do not always align perfectly – imperfect matches can occur in transfer function space because the parametric model is optimized to match model output to data in the time domain (not frequency domain as in Figure 3). The second peak in the short direction is a result of the bracing natural frequency and is not considered significant for this analysis. Table 2 summarizes first-mode parameters recovered from application of these system identification procedures.

Table 2 shows results of analyses of anticipated period lengthening and foundation damping effects using Eqs. (3) to (6) with impedance function solutions for uniform halfspaces adapted to field conditions per NIST (2012). Nonlinear effects were approximated in an equivalent-linear sense using an estimate of shear strain as the ratio of PGV to the near-surface soil shear wave velocity. The two analytical solutions for foundation damping in this test case are generally similar, with the damping from Eq. (5) being smaller than that from Eq. (4). When many additional tests are considered, including cases from the literature, we find that the smaller damping from Eq. 5 is generally in better accord with field performance data when $h/(V_s T) > 0.1$, which represents the conditions where SSI effects are most significant (NIST, 2012).

Table 1. Input-output pairs to evaluate flexible- and fixed-base modal parameters. (Tileylioglu, 2008)

Base Fixity	Input	Output
Flexible-Base	F_s/m_s	$\ddot{u}_f + h\ddot{\theta}_f + \ddot{u}_s$
Fixed-Base	$F_s/m_s - \ddot{u}_f - h\ddot{\theta}_f$	$\ddot{u}_s$

Note: F_s = shaker force, m_s = structural mass, h = height of the center of m_s, $\ddot{u}_f$ = foundation horizontal acceleration, $\ddot{\theta}_f$ = foundation angular acceleration, and $\ddot{u}_s$ = top mass acceleration associated with structural stiffness/damping.

Table 2. Foundation damping results from forced vibration testing of test structure in unbraced configuration.

Shaking Direction	Observed Data(1)		Results from Theoretical Models(2)		
	Period Lengthening, $\tilde{T}/T$	Foundation Damping, β_f	Period Lengthening	Equation 4 Result	Equation 5 Result
Long direction	1.33	10.0%	1.34	10.5%	9.1%
Short direction	1.65	5.3%	1.55	6.6%	6.1%

Note:
(1) Back-calculated using Eq. 1, n = 2, fixed-base and flexible-base periods and damping from Figure 3.
(2) Solved using Pais and Kausel impedance functions as presented in NIST 2012and β_s = 9.0 (long direction) and β_s = 7.1 (short direction).

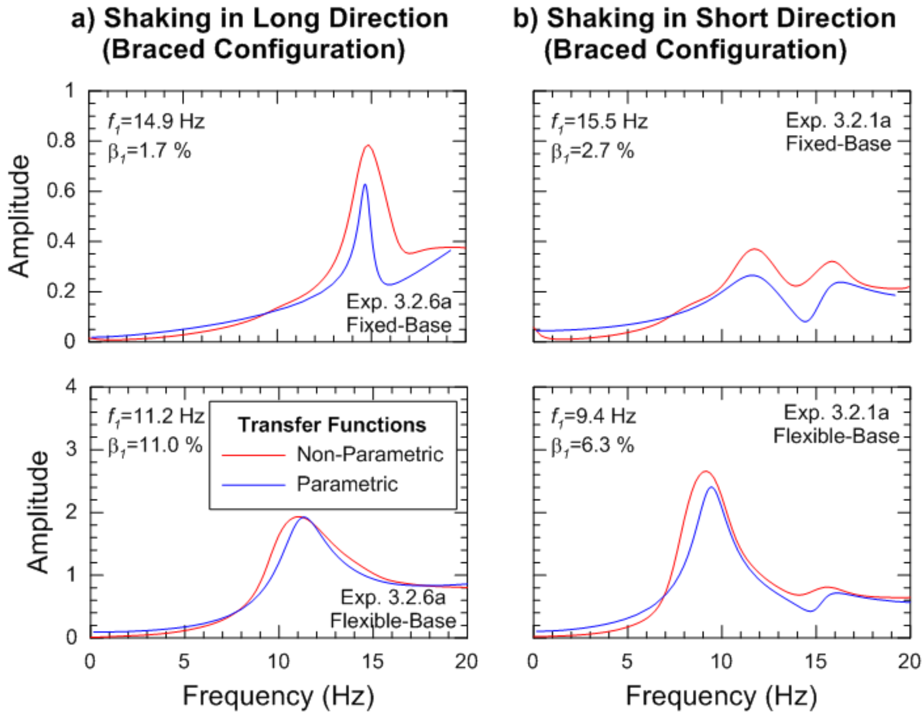

Figure 3. Transmissibility functions from parametric and non-parametric system identification using response history data from large-force forced vibration testing.

Conclusions

Two expressions for foundation damping (Eq. 4 and Eq. 5) have been presented that were derived from first principles elsewhere (Givens et al. 2015). An advantage of the present solution relative to classical solutions in the literature is that foundation damping is expressed as the sum of readily understandable terms representing hysteretic and various forms of radiation damping. On theoretical grounds, there is no clear benefit of one of the aforementioned foundation damping solutions over the other. A practical benefit of Eq. 4 is that it is expressed entirely in terms of real-valued variables, whereas Eq. 5 includes complex variables that are converted to real numbers using absolute values.

Forced vibration tests were performed on a portable steel column structure with concrete top and bottom slabs to measure SSI effects. The test structure was reconfigurable to provide alternate structure stiffnesses and the tests were performed at three test sites with different soil conditions. Acceleration, displacement, and foundation pressure data was recorded and archived at the Network for Earthquake Engineering Simulations Research (NEESR) website as project NEES-2008-0637. Data from these tests are useful for inference of foundation stiffness and damping in the form of impedance functions. In this paper we focus on foundation damping for a single case. Results are useful to establish general compatibility between model predictions and data.

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