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Simple Schemes of Multi–anchored Flexible Walls Dynamic Behavior

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ABSTRACT

Simple schemes of dynamic behavior of multi – anchored flexible retaining walls are examined in order to capture the typical frequencies involved during strong motion earthquakes. In particular multi – anchored retaining walls are sketched with clamped ends, due to its own anchorages, three rigidity springs (the wall's and the two anchored soil wedges, whose inertial effects are assumed to be separated) and two masses as two earth wedges are involved.

Introduction

In an effort to better understand the actual behavior of a cantilever flexible wall to sustain a deep excavation during a strong motion earthquake and following the results of an important research reported in Al Atik and Sitar (2010), Garini (2014) hypothesized that the Mononobe – Okabe seismic wedge should further be divided in two sub wedges that move out of phase. In particular Garini found that the point of application of the seismic active thrust should indeed be determined by this assumption, as the two wedges are individuated by the their own dynamic equilibrium.

Indeed Al Atik and Sitar (2010) state the following main conclusions:

1. There seems to be no basis for the currently accepted position of the dynamic earth pressure force in dynamic L.E.A. at 0.6 to 0.67 H and, instead, the point of application should be at 1/3 H;
2. Maximum dynamic earth pressures and maximum wall inertial forces do not tend to occur simultaneously. As a result, the current design methods based on the Mononobe Okabe theory were found to significantly overestimate dynamic earth pressures and moments.
3. Seismic earth pressures on cantilever retaining walls can be neglected at accelerations below 0.4 g.

Garini (2014) gave an interpretation of the findings reported in Al Atik & Sitar (2010) regarding the seismic active thrust on cantilever flexible walls resorting to correct rational mechanics statements i.e. the application of the seismic force in the centroid at 2/3H.

In this paper I will focus on multi – anchored flexible retaining walls subjected to a strong motion earthquake and also in this case I assume that, due to the typical mechanism of the thrust wedge that has an outer shallow/softer zone and an internal more confined zone, it's possible to consider two independent sub – wedges behind the multi - anchored flexible wall.

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Due to the typical constraints of multi – anchored flexible walls, this assumption leads to sketch a coupled oscillators mechanical two masses and three rigidity springs system.

The Coupled Oscillators Mechanical System

In the Coupled Oscillator mechanical System we have the sketch reproduced in Figure 1.

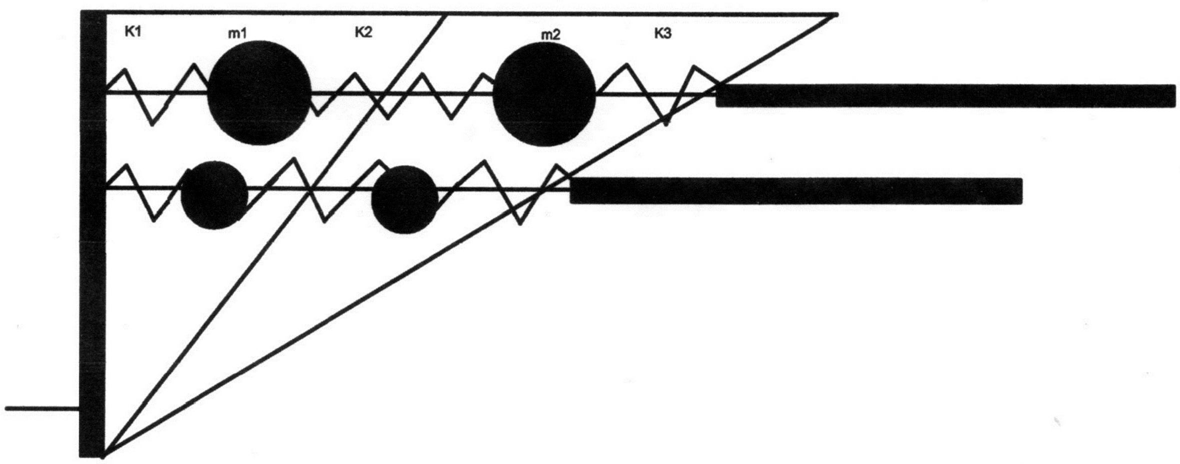

Figure 1. A double anchored flexible wall sketched with 2 coupled oscillators with their own two masses and three rigidity springs representing the Mononobe Okabe seismic active wedge divided in two sub wedges, with clamped ends in the wall and the tie back foundations.

This system has the following two natural frequencies (see Appendix):

$$\omega_{1,2} = \sqrt{\frac{\left[\left(\frac{K_1+K_2}{m_1}\right)+\left(\frac{K_2+K_3}{m_2}\right)\right] \pm \sqrt{\left[\left(\frac{K_1+K_2}{m_1}\right)+\left(\frac{K_2+K_3}{m_2}\right)\right]^2 - 4\left[\frac{(K_1+K_2)(K_2+K_3)-K_2^2}{m_1 \, m_2}\right]}}{2}} \qquad (1)$$

where m_i (i=1,2) are the sub wedge i mass and for each half and double – half sub – wedge (for i = 1,2,3), we have:

$$K_i = E_i A_i / l_i \qquad (2)$$

while notice that for $K_3 = 0$,we have the following:

$$\omega_{1,2} = \sqrt{\frac{\left[\left(\frac{K_1+K_2}{m_1}\right)+\left(\frac{K_2}{m_2}\right)\right] \pm \sqrt{\left[\left(\frac{K_1+K_2}{m_1}\right)+\left(\frac{K_2}{m_2}\right)\right]^2 - 4\left[\frac{K_1 K_2}{m_1 m_2}\right]}}{2}} \qquad (3)$$

It’s clear from physics that the coupled oscillators have one antisymmetrical mode of vibration (out of phase) leaving unalterable the centroid position and one symmetrical mode of vibration (in phase) leaving unalterable the relative distance between the two masses.

The out of phase mode, namely the antisymmetrical mode of vibration, when excited by the strong motion, considering in this case, equation (3) valid for the wall mass and the entire Mononobe Okabe soil wedge mass, well explain the findings of Al Atik and Sitar (2010), so this means that the predominant period from 0.2 to 0.62 s (ω = 15.32 rad/sec in average) in their work captures this typical frequency.

Equation (3) can simulate the dynamic behavior of a cantilever flexible wall, either as above indicated for the work of Al Atik and Sitar (2010) or for the soil sub wedges as reported in Garini(2014).

Equation (2) applies at each anchorage system as the friction alongside the mass around anchorage cancel each other.

More Accurate Coupled Oscillators Behavior

So far I have assumed that spring stiffness K_i, were behaving in a linear fashion with the same value both in tension and compression.

A more precise law for soils may be as that indicated in the hysteresis loop of Figure 2, where, during soil stress strain compression and extension along the anchorage line, we can distinguish between compression and tension stress respectively; so as in the sketches depicted in Figure 3 in general we will alternatively have, with obvious significance of the suffixes :

K_{2T}, K_{1T} associated with K_{3C};

K_{2C}, K_{1C} associated with K_{3T};

K_{1C}, K_{2T} associated with K_{3C};

K_{1T}, K_{2C} associated with K_{3T};

And this means that we have to consider a quadruple series of the two natural frequencies according to equation (1).

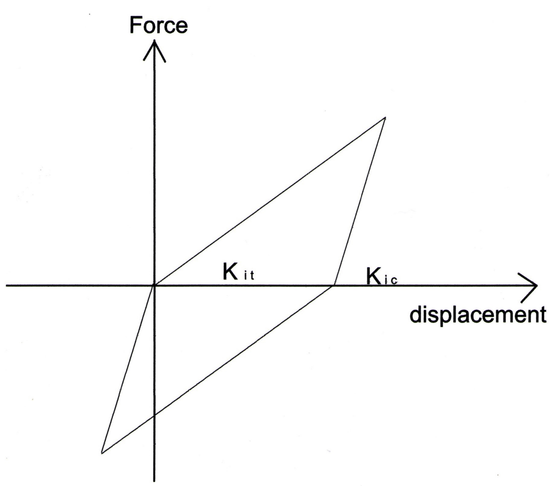

Figure 2. Hysteretic soil behaviour.

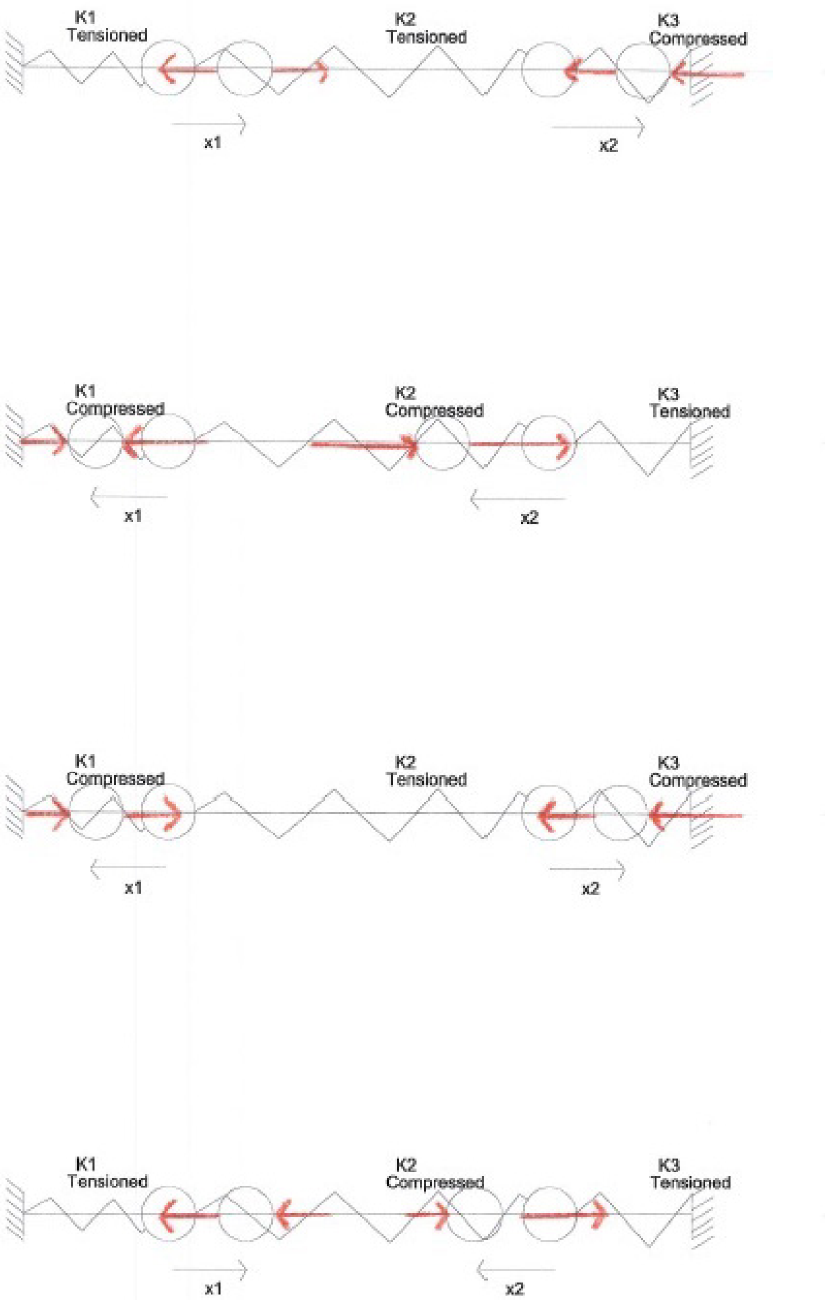

Figure 3. Quadruple scheme to evaluate spring stiffness either in tension or in compression.

Example Case

We can consider the simplest case with the same spring stiffness either in compression or in tension for a double anchorage at 1 and 3 m of depth, and an excavation height of 6m with the Young Modulus $E = 2(1+0,3)\ 5.3\ 10^4$KPa as in Al Atik and Sitar(2010) and individuate the two sub wedges for example from the second case described in Garini (2014), where we get $l_{subwedge\ 1} = 1.29$ m and $l_{wedge} = 4.02$ m. Considering the portions with $A = 2\ m^2$ of the sub – wedges involved ,we get the results showed in Table 1:

Table 1. Anchorage Coupled Oscillators natural frequencies Calculation.

Anchorage	K_1(kN/m)	K_2(kN/m)	K_3(kN/m)	$m_1(kg_m)$	$m_2(kg_m)$	ω_1(rad/s)	ω_2(rad/s)
1	512181	164363	242034	3730	7893	6.69	13.72
2	853635	273939	403390	2238	4736	11.15	22.86

So we have for shallow anchorages low natural frequencies namely high natural periods and vice versa for deeper anchorages and then corresponding probable resonance effects.

It's important to notice that these results correspond with the work of Steedman and Zeng (1990), where K_{AE} is shown as a function of the adimensional ratio H/TV_S, which is the ratio of time for a wave to travel the full excavation height to the period of lateral shacking.

Indeed, for a given excavation depth H, the higher is the natural period and so the corresponding probable strong motion resonance effects period in a seismic amplification behavior, the lower the ratio H/TV_S and the higher the coefficient K_{AE} namely the dynamic force on the wall and so the mass involved.

Moreover this is particularly true for higher p.g.a., that is what I am considering due to the above said reported findings of Al Atik and Sitar (2010) and Garini(2014).

Conclusions

An interpretation of the dynamic behavior behind a multi – anchored flexible wall has been given having recourse to a simple coupled oscillators mechanical system, for which the natural frequencies have been recalculated even in the case of two hanging masses to simulate in this case a cantilever flexible wall.

These considerations involve the importance of lower natural frequencies in the shallower anchorages and vice versa for the deeper anchorages.

Of course this means that as an ordinary coupled oscillator we have one antisymmetrical (out of phase) and one symmetrical mode of vibration (in phase) so to rightly justify the dynamical behavior finding of Al Atik and Sitar (2010), where it was argued a different oscillation between the wall and the soil, which in Garini(2014), who assumed a different oscillation between and outer soil wedge and an inner constrained sub wedge, found an interesting dynamic equilibrium explanation.

In particular it is found a good agreement of this mechanical interpretation of the soil behavior behind the wall with the dependence of the seismic coefficient K_{AE} with the period of the seismic excitation as stated previously by Steedman and Zeng (1990).

References

Al Atik L., Sitar N. Seismic Earth Pressures on Cantilever Retaining Structures. *Journal of Geotechnical and Geoenvironmental Engineering* 2010; **136**: 1324-1333.

Garini A.D. A New Approach for Seismic Active Thrusts in deep Excavations. *Geotechnical Aspects of Underground Construction in soft Ground* 2014; 55- 59.

Steedman R. S., Zeng X. The influence of phase on the calculation of pseudo-static earth pressure on a retaining wall. *Géotechnique* 1990; **40**: 103-112.

Appendix

From the sketch in Figure 1 we have:

$$m_1\ddot{x}_1 = -K_1x_1 + K_2(x_2 - x_1) \quad (4)$$

$$m_2\ddot{x}_2 = -K_2(x_2 - x_1) - K_3x_2 \quad (5)$$

So if we write:

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (6)$$

$$A = \begin{pmatrix} \frac{K_1+K_2}{m_1} & -\frac{K_2}{m_1} \\ -\frac{K_2}{m_2} & \frac{K+K_3}{m_2} \end{pmatrix} \quad (7)$$

$$[A](X) = \omega^2(X) \quad (8)$$

We find the natural frequencies given in Equation (1), solving the followings:

$$det(A - \omega^2\mathbf{1}) = 0 \quad (9)$$

$$\left(\frac{K_1+K_2}{m_1} - \omega^2\right)\left(\frac{K_2+K_3}{m_2} - \omega^2\right) - \frac{K_2^2}{m_1m_2} = \mathbf{0} \quad (10)$$

$$\omega^4 - \omega^2\left(\frac{K_1+K_2}{m_1} + \frac{K_2+K_3}{m_2}\right) + \left(\frac{K_1+K_2}{m_1}\right)\left(\frac{K_2+K_3}{m_2}\right) - \frac{K_2^2}{m_1m_2} = \mathbf{0} \quad (11)$$