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On the Estimation of Earthquake Induced Ground Strains from Velocity Recordings: Application to Centrifuge Dynamic Tests

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ABSTRACT

Soil dynamic properties can be determined in centrifuge tests through the shear stress-strain cycles obtained from the accelerations recorded within the soil model. Despite its simplicity, a significant source of uncertainty of the method has been recognized to lie in the determination of the strain amplitude. Recently, a method was proposed to compute the mobilized shear modulus and damping ratio using the nonlinear fit of experimental transfer functions with the analytical expression of the amplification function for a viscoelastic layer on a rigid base (Conti & Viggiani, 2012). According to this method, the corresponding shear strain is a function of the particle velocity and of the shear wave velocity. However, the equation proposed by the Authors strictly holds only for almost mono-frequency signals. This paper addresses the problem of using velocity measurements to compute the shear strains induced by vertically propagating shear waves in a uniform soil layer. Theoretical results are applied to the interpretation of centrifuge dynamic tests.

Introduction

The non-linear and hysteretic behaviour of soils is generally described by the shear modulus, *G*, and the damping ratio, *D*, and their variation with shear strain level, γ. As amplification phenomena inside a soil layer depend strongly on the shear stiffness and the damping ratio mobilised during the earthquake, the $G(\gamma)$ and $D(\gamma)$ curves are crucial ingredients to determine the seismic response of geotechnical systems in soil-structure interaction problems.

Zhegal *et al*. (1995) showed that *G* and *D* can be determined from *in-situ* acceleration time histories recorded during real earthquakes. The method is based on the evaluation of shear stress-strain cycles obtained from down-hole acceleration time histories recorded at different depths in the soil at instrumented test sites. The same method has been applied to the acceleration time histories recorded in centrifuge models (Zeghal *et al*., 1998; Brennan *et al*., 2005). In this case, the greatest source of uncertainty lies in the determination of the shear strain amplitude (Zeghal *et al*. 1998) which may be affected by large and not easily definable errors.

More recently, Conti & Viggiani (2012) suggested to compute the mobilised shear modulus and damping ratio using the nonlinear fit of experimental transfer functions obtained at different depths in centrifuge models. Moreover, they showed that the free-field shear strain induced into the soil model by a vertically propagating shear wave cannot be computed using the theory of wave propagation in homogeneous isotropic elastic media, *i.e*. $\gamma = v/V_S$ (v = particle velocity, V_S = shear wave velocity), due to multiple refraction and reflection of waves generated by boundary conditions. However, the equation proposed by the Authors was derived for the sole

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case of a uniform undamped soil layer and, also, strictly holds only for the almost mono-frequency signals applied in the centrifuge dynamic tests analysed in their paper.

This work addresses the problem of using velocity measurements to compute the shear strains induced by vertically propagating shear waves in a uniform soil layer. After recalling the main steps of the data reduction procedure proposed by Conti & Viggiani (2012), the exact point wise relation between particle velocity and shear strain is derived through the definition of a suitable transfer function, under the assumption of isotropic viscoelastic behaviour for the soil. Theoretical results are applied to the interpretation of centrifuge dynamic tests.

Evaluation of Soil Dynamic Properties in Centrifuge Tests

According to Conti & Viggiani (2012), the shear modulus and the damping ratio mobilised during an earthquake at mid-height of the centrifuge model can be estimated from the amplification function of the soil layer *A*(f), obtained from the free-field accelerations measured at the bottom and close to the surface. The proposed method includes three steps: (i) identification of the range of frequencies significant for both signals, and calculation of the experimental amplification function within this range; (ii) non-linear interpolation of the experimental curves with the analytical expression of the amplification function of a visco-elastic layer on a rigid base and determination of *G* and *D*; (iii) calculation of the mobilised shear strain at mid-height of the sand layer. The procedure for data reduction will be recalled briefly in the following, with reference to one of the test performed by the Authors on dry sand (Test CW6).

Step 1: Identification of Relevant Frequencies

The range of frequencies that are common to signals $x(t)$ and $y(t)$, registered at the bottom and close to the top of the sand layer respectively, can be identified by the cross-power spectrum $G_{xy}(f)$ of the two signals, given by:

$$G_{xy}(f) = X(f) \cdot Y^{*}(f) \tag{1}$$

where $X(f)$ is the Fourier spectrum of $x(t)$ and $Y^*(f)$ is the complex conjugate of the Fourier spectrum of $y(t)$. In particular, the frequencies that are significant for both signals can be identified as those at which the amplitude of the normalised cross-power spectrum is greater than a threshold ($TOL = 10^{-4} \div 10^{-5}$). The procedure is illustrated in Figure 1 which shows, for one of the earthquakes applied in the test, (a,b) the Fourier spectra of the accelerations recorded in free field conditions, (c) the normalised cross-power spectrum of the two signals, and (d) the experimental amplification function. In Figure 1(d), the continuous line is the original amplification function for all frequencies in the Fourier spectra, while the dots represent the subset of the amplification function for those frequencies where the normalised cross power spectrum of *x* and *y* is larger than the prescribed threshold.

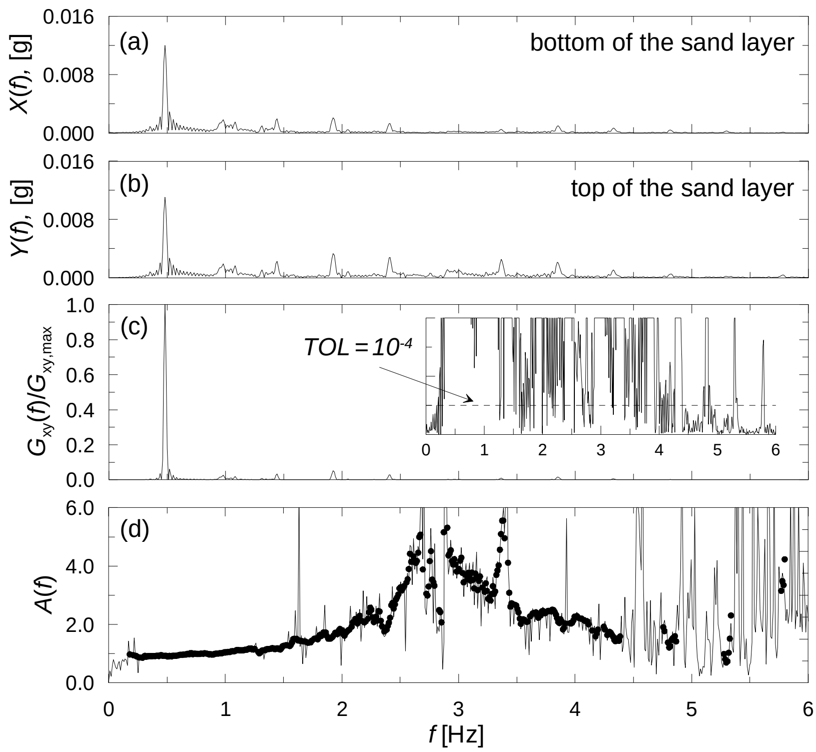

Figure 1. Fourier spectrum of the accelerations at (a) the bottom and (b) on top of the sand layer and corresponding (c) normalised cross-power spectrum and (d) amplification function (Conti & Viggiani, 2012 - Test CW6, EQ1)

Step 2: Computation of Mobilized Shear Modulus and Damping

The experimental amplification function $A(f)$ is given by:

$$A(f) = \frac{|Y(f)|}{|X(f)|} \tag{2}$$

where $|X(f)|$ and $|Y(f)|$ are the amplitudes of the Fourier spectra of $x(t)$ and $y(t)$, calculated for the range of frequencies in common to the two signals. The analytical expression of the amplification function for a visco-elastic soil layer on a rigid base, defined at a depth z in the sand layer, is given by:

$$A(f) = \frac{\sqrt{\cos^2 kz + (Dkz)^2}}{\sqrt{\cos^2 kH + (DkH)^2}} \tag{3}$$

where $k = 2\pi f/V_S$ is the wave number. For a given depth z, the amplification function depends on the shear wave velocity and the damping ratio of the sand layer. Therefore, it is possible to determine the mobilised V_S and D by means of a nonlinear least-squares interpolation of the experimental data through Equation (3). The shear modulus mobilised during each earthquake is then calculated as $G = \rho V_S^2$. As an example, Figure 2 shows the experimental data and the best fitted amplification functions for all the earthquakes of Test CW6.

Step 3: Computation of Mobilized Shear Strain

According to Conti & Viggiani (2012), the maximum shear strain mobilised at a given depth, z, is related to the maximum horizontal ground velocity and to the shear wave velocity as:

$$\gamma_{max}(z) = \frac{v_{max}(z)}{V_S} \tan kz \tag{4}$$

where the wave number k is computed using the mean quadratic frequency of the acceleration signal. It follows that the maximum shear strain mobilised at mid-height of the sand layer ($z = H/2$) can be estimated from the maximum horizontal velocity obtained integrating the horizontal acceleration registered at mid-height. However, Equation (4) refers to the case of a uniform undamped soil layer and, also, strictly holds only for almost mono-frequency signals. In the next section, a more refined solution is derived under the assumption of isotropic viscoelastic behaviour for the soil.

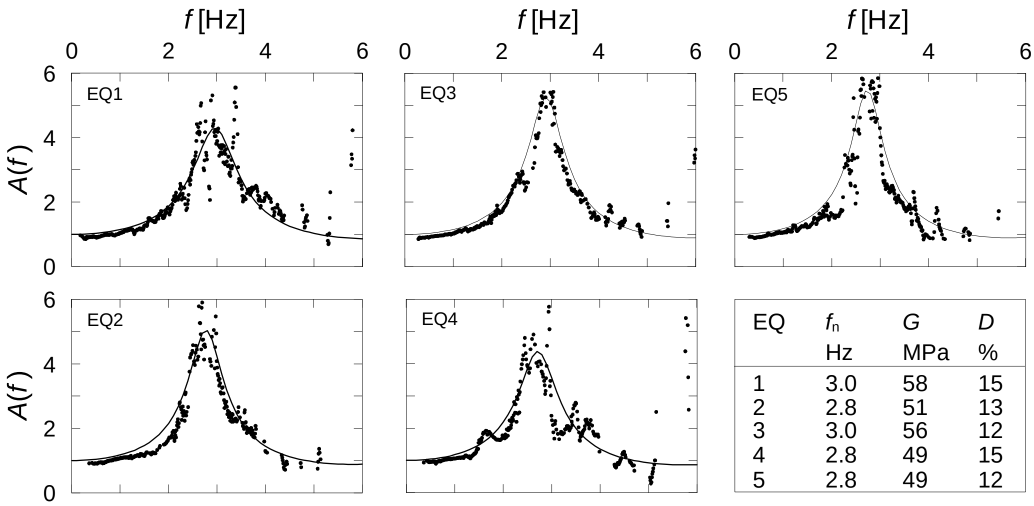

Figure 2. Experimental data and best-fitted amplification functions between the bottom and the top of the sand layer (Conti & Viggiani, 2012 - Test CW6).

Computation of Free Field Shear Strains from Velocity Recordings

In the following, the simple case of a pure harmonic input motion will be considered first, and then the result will be extended to the general case of real earthquakes or multi-frequency input signals.

Case I: Simple Harmonic Input Motion

The problem under consideration consists of a uniform layer of isotropic, linear visco-elastic material overlying a rigid base, corresponding to which a simple harmonic input motion of angular frequency ω is applied:

$$u(H,t) = Ue^{i\omega t} \tag{5}$$

For the problem at hand, the solution of the equation of motion is (Kramer, 1996):

$$u(z,t) = 2Ae^{i\omega t}\cos k^* z \tag{6}$$

where $k^* = \omega/V_S^*$ is the complex wave number, V_S^* is the complex shear wave velocity, $G^* = G(1+2iD)$ is the complex shear modulus, and D is the damping ratio. By differentiating Equation (6) with respect to t and z, we get:

$$v(z,t) = 2i\omega Ae^{i\omega t}\cos k^* z \tag{7}$$

$$\gamma(z,t) = -2k^* Ae^{i\omega t}\sin k^* z \tag{8}$$

It follows that the shear strain and the particle velocity at a given depth within the soil layer are related by the equation:

$$\gamma(z,t) = F(z,\omega)\cdot v(z,t) \tag{9}$$

where

$$F(z,\omega) := \frac{i}{V_S^*}\frac{\sin k^* z}{\cos k^* z} \tag{10}$$

is the strain transfer function, depending on the mechanical and physical properties of the soil layer (V_S, D), on the frequency content of the input motion (ω), and on the soil depth (z). Therefore, the maximum shear strain at a given depth can be computed as

$$\gamma_{\max}(z) = |F(z,\omega)|\cdot v_{\max}(z) \tag{11}$$

where, for small values of the damping ratio, $|F(z,\omega)|$ is given by (see Appendix):

$$|F(z,\omega)| = \frac{1}{V_S}\frac{\sqrt{\sin^2 kz + (Dkz)^2}}{\sqrt{\cos^2 kz + (Dkz)^2}} \tag{12}$$

As an example, Figure 3 shows the absolute value of the strain transfer function, $|F(z,\omega)|$, multiplied by the shear wave velocity, V_S. Local maxima of the function are attained for $kz = (n+1/2)\pi$, with $n = 0,1,2,\ldots,\infty$ or, equivalently, for $z/H = \omega_n/\omega = f_n/f$, where $\omega_n = V_S/H\cdot(n+1/2)\pi$ are the natural frequencies of the soil stratum. It is evident from the plot that each component in the velocity signal would be filtered differently depending on both the depth, z, and the frequency, f.

Case II: Earthquake Input Motion

As far as real earthquakes or multi-frequency signals are concerned, a standard two-step procedure can be followed to compute the soil shear strain from a recorded velocity time history at a given depth, that is: (i) decompose the velocity signal by means of its Fourier transform

$$v(z,t) = \sum_{n=-\infty}^{+\infty} v_n^* e^{i\omega_n t} \tag{13}$$

and (ii) exploit the linearity of the system and the principle of superposition to compute the shear strain as:

$$\gamma(z,t) = \sum_{n=-\infty}^{+\infty} F(z,\omega_n) v_n^* e^{i\omega_n t} \tag{14}$$

Finally, the maximum shear strain is simply $\gamma_{\max}(z) = \max \gamma(z,t)$.

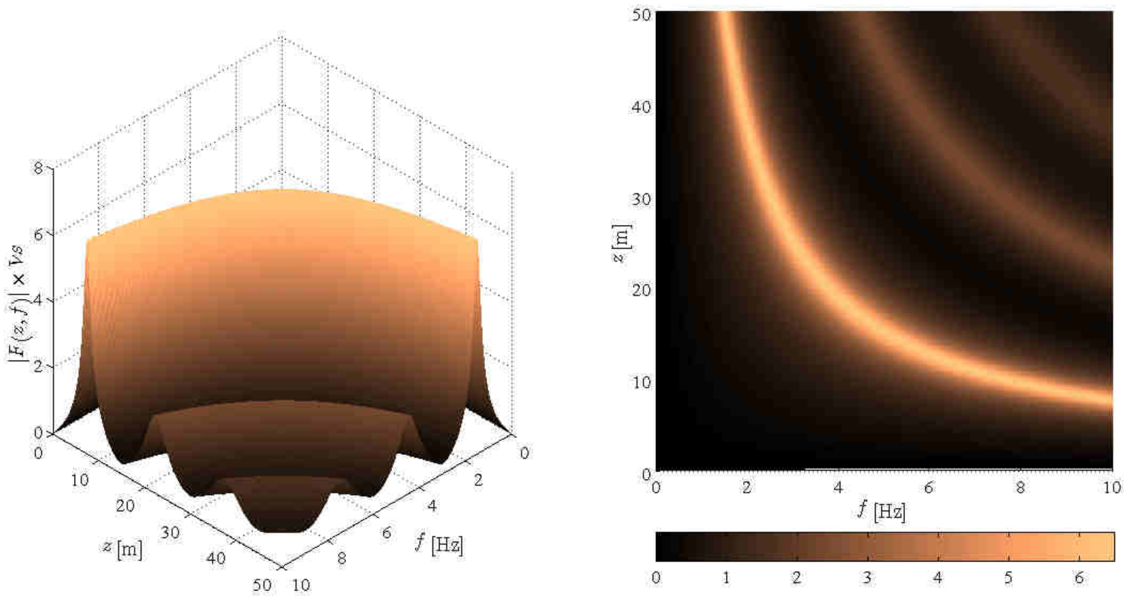

Figure 3. Strain transfer function (V_S = 300 m/s, γ= 18 kN/m^3, D = 0.1, H = 50 m).

Discussion of Results and Applications

Figure 4 shows the normalised shear modulus and the damping ratio computed using the nonlinear fit of experimental transfer functions for all the earthquakes of the nine centrifuge tests described in Conti & Viggiani (2012). Maximum shear strain at mid-height of the sand layer is computed using both Equation (4) and the more rigorous solution proposed in this paper. For comparison, Figure 4 shows also the empirical upper and lower bounds given by Seed & Idriss (1970) for dry sand (shaded area) and the experimental curves (dashed lines) suggested by Vucetic & Dobry (1991) for cohesionless soils (plasticity index, PI = 0). These curves must be considered as average responses since they were derived from a variety of test procedures and tested sands. The maximum difference between the two solutions is about 100%, computed at relatively small value of the mobilised shear strains ($\gamma \sim 0.05 \div 0.1$ %), but no relevant differences exist in the experimental curves. This observation is mainly due to the fact that the accelerations applied in the tests are almost mono-frequency signals and, hence, the approximate procedure proposed by Conti & Viggiani (2012) induces no significant errors in the estimation of the mobilised shear strain. As far as the shear modulus reduction curve is concerned, physical model results obtained using both methods are in very good agreement with literature data. On the other hand, a more dispersed trend is observed in terms of damping ratio. In fact, in this case centrifuge data are generally higher than values reported in the literature and show a broader scatter compared to the shear modulus values, which can be partly attributed to the larger percentage errors in the estimates of *D* (Conti & Viggiani, 2012). Note that a certain amount of scatter in damping ratios has been observed by other authors (Brennan *et al.*, 2005; Rayhani & El Naggar, 2008), for different soils and stress states.

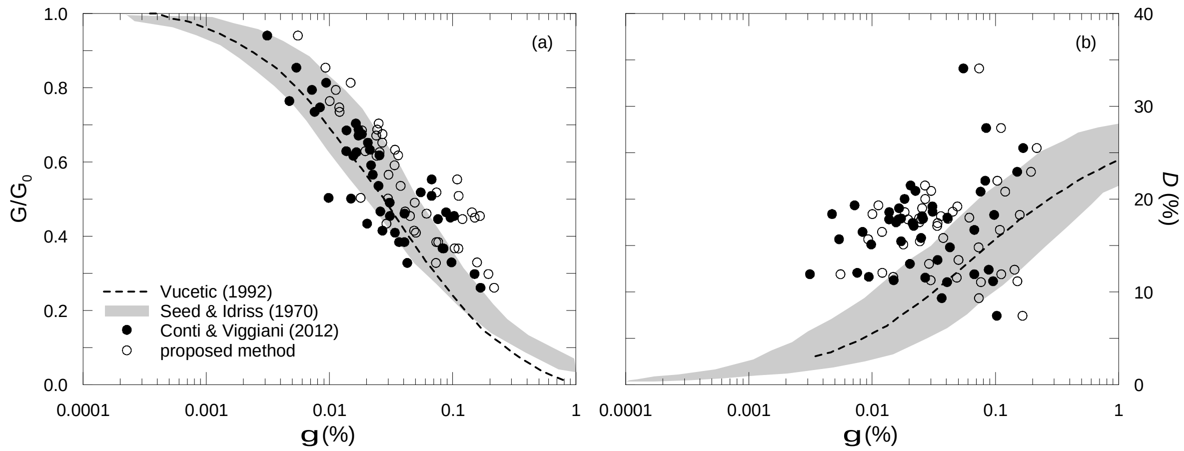

Figure 4. (a) Shear modulus and (b) damping ratio from centrifuge data.

Conclusions

This note focused on the problem of using velocity measurements to compute the shear strains induced by vertically propagating shear waves in dynamic centrifuge tests. Specifically, the exact point wise relation between particle velocity and shear strain was derived through the definition of a suitable transfer function, under the assumption of isotropic viscoelastic behaviour for the soil. The main motivation for this work was the new method proposed by Conti & Viggiani

(2012) to compute the mobilised shear modulus and damping from the non-linear fit of experimental transfer functions.

Theoretical results were applied to the interpretation of nine tests carried out on small scale models of dry sand. In general, the experimental data compare very well with the literature data in terms $G/G_0(\gamma)$ curve, while a more dispersed trend is observed for $D(\gamma)$.

The relation proposed for the estimation of mobilised shear strains could be applied for the interpretation of centrifuge dynamic tests when either real earthquakes or multi-frequency signals are applied to the soil models.

Acknowledgments

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Appendix

For small values of the damping ratio ($D << 1$) we have $V_S^* \approx V_S(1+iD)$ and $k^* \approx k(1-iD)$. Exploiting the identities:

$$|\cos(x+iy)|^2 = \cos^2 x + \sinh^2 y \tag{A.1}$$

$$|\sin(x+iy)|^2 = \sin^2 x + \sinh^2 y \tag{A.2}$$

it is possible to write:

$$|\cos k^* z| \approx |\cos kz(1-iD)| = \sqrt{\cos^2 kz + \sinh^2 Dkz} \tag{A.3}$$

$$|\sin k^* z| \approx |\sin kz(1-iD)| = \sqrt{\sin^2 kz + \sinh^2 Dkz} \tag{A.4}$$

Finally, since $|V_S^*| \approx V_S$ and $\sinh Dkz \approx Dkz$ for small values of *D*, Equation (12) is proved.