6th International Conference on Earthquake Geotechnical Engineering
1-4 November 2015
Christchurch, New Zealand

Earthquake Effects on Structures Embedded in Saturated Granular Deposits

A. Cividini[1], G. Gioda[2]

ABSTRACT

The studies on the dynamic analysis of saturated soils led to various numerical approaches that involve different assumptions, different governing equations and also different sets of free variables. The relatively complex mathematical structure of the problem does not permit a straightforward evaluation of the consequences of these assumptions and, hence, makes the choice of the most appropriate numerical approach somewhat controversial. Here the complete formulation of dynamic two-phase problems is first summarized, under assumptions which seem acceptable in the geotechnical engineering context. Then, two finite element approaches are derived on this basis, the latter of which permits reducing the number of free nodal variables with respect to the first one. Finally, the results obtained in the solution of two benchmark problems are presented and commented upon.

Introduction

The solution of geotechnical problems involving saturated two phase-soils requires the simultaneous analyses of the seepage flow and of the effective stress distribution within the soil skeleton. In quasi static conditions, under an acceleration field constant with time (i.e. the gravity field), the literature provides exhaustive theoretical bases and broadly accepted methods for the numerical analysis of seepage and of the coupled effective stress-flow problem, e.g. Desai (1976), Sandhu & Wilson (1969), Zaman et al. (2000).

In dynamic conditions however, e.g. during earthquakes, the analysis of seepage becomes less straightforward since recourse cannot be made to the usual concept of hydraulic head (Bear, 1988; Bird et al., 2007). This led to various numerical approaches for dynamic coupled problems that involve different assumptions, different governing equations and different sets of free variables (Zienkiewicz & Shiomi 1984; Cividini & Pergalani, 1994; Zienkiewicz et al., 1999).

The relatively complex mathematical structure of the problem does not permit a straightforward evaluation of the consequences of these assumptions and, hence, makes the choice of the most appropriate numerical approach somewhat controversial. This suggested undertaking a study on the coupled dynamic analysis of saturated granular deposits.

[1]Professor, Dept. of Architecture, Built environment and Construction engineering, Politecnico di Milano, Milano, Italy, annamaria.cividini@polimi.it
[2]Professor, Dept. of Architecture, Built environment and Construction engineering, Politecnico di Milano, Milano, Italy, giancarlo.gioda@polimi.it

Its initial part, limited to dynamic seepage flows, was presented in Stucchi et al. (2010) and in Cividini & Gioda (2014). Here the complete formulation of dynamic two-phase problems is summarized, observing some difference in the final finite element equations with respect to those of other formulations presented in the literature. Then a simplified formulation, worked out in order to reduce the number of the free nodal variables, is recalled. The details of derivations are rather lengthy and are omitted here for sake of briefness. They will be presented in a parallel paper (Cividini & Gioda, 2015).

Two test problems are considered. The first one concerns the dynamic effects on a vertical rigid wall confining a water reservoir. The results obtained with the first formulation are compared with the closed form solution proposed by Westergaard (1933). The example was also solved with the "reduced mixed formulation" proposed by Zienkiewicz & Shiomi (1984). Then, the two formulations considered here are applied to the solution of a second illustrative problem concerning the dynamic behaviour of a flexible retaining wall. The comparison of their results permits drawing some preliminary conclusions on the accuracy of the simplified approach with respect to the "complete" one.

Governing Equations

The equations necessary to describe the behavior of the liquid phase (which is denoted by subscript index *L*) are recalled first. They hold under the following assumptions that seem acceptable in the geotechnical context: a Newtonian pore liquid (water) is considered with constant deviatoric viscosity and no volumetric viscosity; the liquid has a constant density and its volumetric deformation linearly depends on the pore pressure; the influence of temperature is neglected; the fluid flow is laminar. They are:

1) Equation of compatibility, relating the strain rate vector $\dot{\boldsymbol{\varepsilon}}_L$ to the relative (with respect to the skeleton) discharge velocity $\boldsymbol{w}$ and to the skeleton velocity.

2) Constitutive relationship, expressing the stresses $\boldsymbol{\sigma}_L$ acting on the liquid phase having pore pressure *p* and accounting for its shear viscosity μ_L.

3) Equation of continuity, enforcing the conservation of the liquid mass, and considering that the bulk modulus B_U depends on the compressibility of water and grains.

4) Equation of motion of the fluid phase, enforcing the momentum balance of the mass of water contained within a fixed unit volume of the porous medium. The terms depend on vectors $\dot{\boldsymbol{w}}$, $\ddot{\boldsymbol{u}}$, $\overline{\boldsymbol{g}}$ collecting, respectively, the components of the relative discharge acceleration; of the skeleton acceleration and of the acceleration of gravity and on the intrinsic permeability matrix $\boldsymbol{K}'$. Note that the quadratic discharge velocity term is neglected because its contribution is marginal in seepage problems.

5) Equation of motion of the two-phase medium written introducing the constitutive matrix of the solid phase $\boldsymbol{D}_S$ and the global constitutive viscosity matrix $\boldsymbol{V}_{SL}$ of the coupled solid and liquid phases.

The dynamic two-phase problem is governed by the system of three differential equations recalled at points 3), 4) and 5), which involves as unknown functions the relative discharge velocity $\boldsymbol{w}$, the skeleton displacements $\boldsymbol{u}$ and the pore pressure *p*.

Boundary Conditions

With reference to confined seepage flows, the saturated porous domain has surface Γ and volume Ω. The surface Γ is subdivided into its impervious part, Γ_w where the relative discharge velocity component normal to it $\overline{w}_n$ vanishes, and its pervious part Γ_p where the pore pressure $\overline{p}$ is known.

The surface Γ can be also subdivided into Γ_u, where the displacements $\overline{\boldsymbol{u}}$ are known, and Γ_σ where the three components of the total surface tractions $\bar{\boldsymbol{t}}$ are imposed.

Finite Element Formulation

Relative discharge velocities $\boldsymbol{w}^e$ and displacements $\boldsymbol{u}^e$ are defined at nodes of the e-th element, while the pore pressure p^e is seen here as an element variable and is defined at the element integration points.

The distributions of relative discharge velocities $\boldsymbol{w}$ and displacements $\boldsymbol{u}$ within the element depend on the interpolation function matrices $\boldsymbol{S}_w^e$, $\boldsymbol{S}_u^e$ and the matrices $\boldsymbol{B}_w^e$, $\boldsymbol{B}_u^e$ contain the space derivatives of the interpolation functions.

The first finite element formulation does not introduce further simplifying assumptions with respect to those already adopted for deriving the governing equations above recalled at points 3), 4) and 5). The finite element form is obtained by writing them and the corresponding boundary conditions in weak form; multiplying them, respectively, by a virtual variation $\delta\boldsymbol{w}$ of the relative discharge velocities and of the displacements $\delta\boldsymbol{u}$ and integrating over the volume Ω and over the relevant part of the surface Γ of an element of the porous medium. This leads to the following set of matrix equations:

$$\begin{bmatrix} \boldsymbol{K}_S^e & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u}^e \\ \boldsymbol{0} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{V}_{Sat}^e & (\boldsymbol{V}_{Lwu}^e)^T \\ \boldsymbol{V}_{Lwu}^e & \boldsymbol{V}_{Lww}^e \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{u}}^e \\ \boldsymbol{w}^e \end{Bmatrix} + \begin{bmatrix} \boldsymbol{M}_{Sat}^e & (\boldsymbol{M}_{Lwu}^e)^T \\ \boldsymbol{M}_{Lwu}^e & \boldsymbol{M}_{Lww}^e \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{u}}^e \\ \dot{\boldsymbol{w}}^e \end{Bmatrix} = \begin{Bmatrix} -\boldsymbol{f}_{up}^e \\ \boldsymbol{f}_{Lp}^e \end{Bmatrix} + \begin{Bmatrix} \overline{\boldsymbol{f}}_{gSat}^e + \overline{\boldsymbol{f}}_t^e \\ \overline{\boldsymbol{f}}_{Lp}^e + \overline{\boldsymbol{f}}_{Lg}^e \end{Bmatrix} \tag{1}$$

Where

$$\boldsymbol{K}_S^e = \int_{\Omega^e} (\boldsymbol{B}_u^e)^T \boldsymbol{D}_S \boldsymbol{B}_u^e d\Omega \qquad \boldsymbol{V}_{Sat}^e = \int_{\Omega^e} (\boldsymbol{B}_u^e)^T \boldsymbol{V}_{SL} \boldsymbol{B}_u^e d\Omega \qquad \boldsymbol{V}_{Lwu}^e = \mu_L \int_{\Omega^e} (\boldsymbol{B}_w^e)^T \boldsymbol{I}_1 \boldsymbol{B}_u^e d\Omega$$

$$\boldsymbol{V}_{Lww}^e = \mu_L \int_{\Omega^e} (\boldsymbol{B}_w^e)^T \boldsymbol{I}_1 \boldsymbol{B}_w^e d\Omega + \mu_L \int_{\Omega^e} (\boldsymbol{S}_w^e)^T (\boldsymbol{K}')^{-1} \boldsymbol{S}_w^e d\Omega$$

$$\boldsymbol{M}_{Sat}^e = \rho_{Sat} \int_{\Omega^e} (\boldsymbol{S}_u^e)^T \boldsymbol{S}_u^e d\Omega \qquad \boldsymbol{M}_{Lwu}^e = \rho_L \int_{\Omega^e} (\boldsymbol{S}_w^e)^T \boldsymbol{S}_u^e d\Omega \qquad \boldsymbol{M}_{Lww}^e = \rho_L \int_{\Omega^e} (\boldsymbol{S}_w^e)^T \boldsymbol{S}_w^e d\Omega$$

$$\boldsymbol{f}_{up}^e = \int_{\Omega^e} (\boldsymbol{B}_u^e)^T \boldsymbol{m} p^e d\Omega \qquad \overline{\boldsymbol{f}}_{gSat}^e = \rho_{Sat} \int_{\Omega^e} (\boldsymbol{S}_u^e)^T \overline{\boldsymbol{g}} d\Omega \qquad \overline{\boldsymbol{f}}_t^e = \int_{\Omega^e} (\boldsymbol{S}_u^e) \boldsymbol{T}_2^T \bar{\boldsymbol{t}}^e d\Omega$$

$$\boldsymbol{f}_{Lp}^e = \int_{\Omega^e} (\boldsymbol{B}_w^e)^T \boldsymbol{m} p^e d\Omega \qquad \overline{\boldsymbol{f}}_{Lp}^e = \int_{\Gamma_p^e} (\boldsymbol{S}_w^e)^T \boldsymbol{T}_1 \boldsymbol{m} \overline{p}^e d\Gamma \qquad \overline{\boldsymbol{f}}_{Lg}^e = \rho_L \int_{\Omega^e} (\boldsymbol{S}_w^e)^T \overline{\boldsymbol{g}} d\Omega$$

Note that vectors $\boldsymbol{f}_{Lp}^{e}$ and $\boldsymbol{f}_{up}^{e}$ depend on the unknown pore pressure distribution within the element. Consequently, also the following equation that represents the finite element form of the continuity equation referred at point 3) is necessary for solution

$$\dot{p}^{e} = B_U(\boldsymbol{m}^T \boldsymbol{B}_u^e \dot{\boldsymbol{u}}^e + \boldsymbol{m}^T \boldsymbol{B}_w^e \boldsymbol{w}^e) \tag{2}$$

This “complete” formulation is referred to in the following as ***u-w*** approach.

A simplified formulation (referred to as ***u*** approach) is obtained taking into account that some terms, in the governing equations cited at points 4) and 5), could be disregarded since their contribution is likely to be marginal (Zienkiewicz et al., 1999). These are the terms that contain the relative discharge acceleration and the second space derivatives of the discharge velocity and of the velocity of the solid phase. Based on these additional assumptions, the governing equation reduces to the following form that does not involve the discharge velocity as a free variable

$$\boldsymbol{K}_S^e \boldsymbol{u}^e + \boldsymbol{V}_{Sat}^e \dot{\boldsymbol{u}}^e + \boldsymbol{M}_{Sat}^e \ddot{\boldsymbol{u}}^e = -\boldsymbol{f}_{up}^e + \overline{\boldsymbol{f}}_{gSat}^e + \overline{\boldsymbol{f}}_t^e \tag{3}$$

Also in this case Equation 2 is necessary for evaluating the pore pressure.

Time Integration Scheme

Let me write Equations 1 and 3 in the compact form expressed by Equation 4, with obvious meanings of symbols. Note that vector $\boldsymbol{b}$ depends on the pore pressure, while $\overline{\boldsymbol{b}}$ is known and depends solely on time *t*.

$$\boldsymbol{Z}_1 \boldsymbol{x}(t) + \boldsymbol{Z}_2 \dot{\boldsymbol{x}}(t) + \boldsymbol{Z}_3 \ddot{\boldsymbol{x}}(t) = \boldsymbol{b}(p,t) + \overline{\boldsymbol{b}}(t) \tag{4}$$

In order to integrate Equation 4 in time, it is assumed that the variation of $\ddot{\boldsymbol{x}}(t)$ within a time increment Δt_i is governed by an a priory chosen interpolation function (Newmark, 1959; Katona & Zienkiewicz, 1985). This leads to the following recursive forms, where $\Delta\ddot{\boldsymbol{x}}_i$ represents the increment of the second derivative at the end of the step and the coefficients β_0 and β_1 depend on the interpolation function adopted for $\ddot{\boldsymbol{x}}(t)$:

$$\boldsymbol{x}_i = \left[\boldsymbol{x}_{i-1} + \Delta t_i \dot{\boldsymbol{x}}_{i-1} + \Delta t_i^2 \ddot{\boldsymbol{x}}_{i-1}/2\right] + \beta_0 \Delta t_i^2 \Delta\ddot{\boldsymbol{x}}_i/2; \ldots \dot{\boldsymbol{x}}_i = \left[\dot{\boldsymbol{x}}_{i-1} + \Delta t_i \ddot{\boldsymbol{x}}_{i-1}\right] + \beta_1 \Delta t_i \Delta\ddot{\boldsymbol{x}}; \ldots \ddot{\boldsymbol{x}}_i = \ddot{\boldsymbol{x}}_{i-1} + \Delta\ddot{\boldsymbol{x}}_i \tag{5}$$

Substitution of Equations 5 into Equation 4 leads to

$$\left[\beta_0 \Delta t_i^2 \boldsymbol{Z}_1/2 + \beta_1 \Delta t_i \boldsymbol{Z}_2 + \boldsymbol{Z}_3\right]\Delta\ddot{\boldsymbol{x}}_i = -\boldsymbol{Z}_1\left[\boldsymbol{x}_{i-1} + \Delta t_i \dot{\boldsymbol{x}}_{i-1} + \Delta t_i^2 \ddot{\boldsymbol{x}}_{i-1}/2\right] - \boldsymbol{Z}_2\left[\dot{\boldsymbol{x}}_{i-1} + \Delta t_i \ddot{\boldsymbol{x}}_{i-1}\right] + \\ -\boldsymbol{Z}_3 \ddot{\boldsymbol{x}}_{i-1} + \boldsymbol{b}(p_i, t_i) + \overline{\boldsymbol{b}}(t_i) \tag{6}$$

Knowing the free variables $\boldsymbol{x}_{i-1}$, their derivatives and the pore pressure at time t_{i-1}, an iterative process is necessary to evaluate them at time t_i:

- Vector $\boldsymbol{b}(p_i, t_i)$ is approximated adopting the values of the pore pressure at time t_i obtained by the previous iteration.

- Vector $\Delta\ddot{\boldsymbol{x}}_i$ is determined by Equation 6, then $\boldsymbol{x}_i$, $\dot{\boldsymbol{x}}_i$, $\ddot{\boldsymbol{x}}_i$ are updated through Equations 5.

- The pore pressure rate $\dot{p}_i$ is evaluated at the integration points of each element by means of Equation 2 and p_i is determined through Equation 5.

- Vector $\boldsymbol{b}(p_i, t_i)$ is updated and the next iteration is carried out.

- The process ends when vector $\boldsymbol{x}(t_i)$ and the pore pressure p_i stabilize.

Considering the small value of the time steps adopted in most dynamic analyses, the iterative process could be avoided in linear analyses by adopting a time marching scheme in which vector $\boldsymbol{b}$ at time t_i is calculated on the basis of the element pore pressure at time t_{i-1}.

Illustrative Examples

Two test examples have been solved through the previously described ***u-w*** "complete" approach. To validate the numerical results, the first example was also solved with the "reduced mixed formulation", or ***u-U*** approach, proposed by Zienkiewicz & Shiomi (1984). The second example is used for investigating the accuracy of the simplified ***u*** approach the results of which are compared with those of the ***u-w*** formulation.

The first example concerns the evaluation of the water pressure distribution on a vertical rigid wall due to a dynamic excitation in the horizontal direction. This problem was first investigated by Westergaard (1933) who provided solutions frequently employed for estimating the effects of earthquakes on dams and on retaining structures in saturated granular soils. Considering that the period of free vibrations T_0 of most dams is appreciably lower than the period of earthquakes T, it can be reasonably assumed that during time all points of the dam have the same horizontal acceleration, which coincides with the one of its base. Westergaard worked out two closed form solutions in plane. The first one neglects the vertical displacement of water, while the second solution takes it into account. The latter one is here adopted for evaluating the dynamic excess water pressure against the wall, i.e. the dynamic water pressure increment with respect to the hydrostatic condition.

The numerical analysis was based on a mesh consisting of 50 four node, quadrilateral isoparametric elements and of 66 nodes (6 of which belong to the vertical wall). The numerical results are compared in Figure 1 with Westergaard solution. Figures 1a and 1b show, respectively, the maximum dynamic excess pressure distribution along the vertical coordinate and the variation with time of the excess pressure at the wall base. In these figures H is the height of the wall, p_{max} is the maximum excess pressure at the base from the closed form solution and T is the period of the sinusoidal excitation.

The ***u-w*** results are also compared with those obtained using the ***u-U*** formulation (Zienkiewicz & Shiomi, 1984). It can be observed that the ***u-w*** approach provides an acceptable approximation of the closed form solution, with an accuracy slightly higher than that of the ***u-U*** formulation.

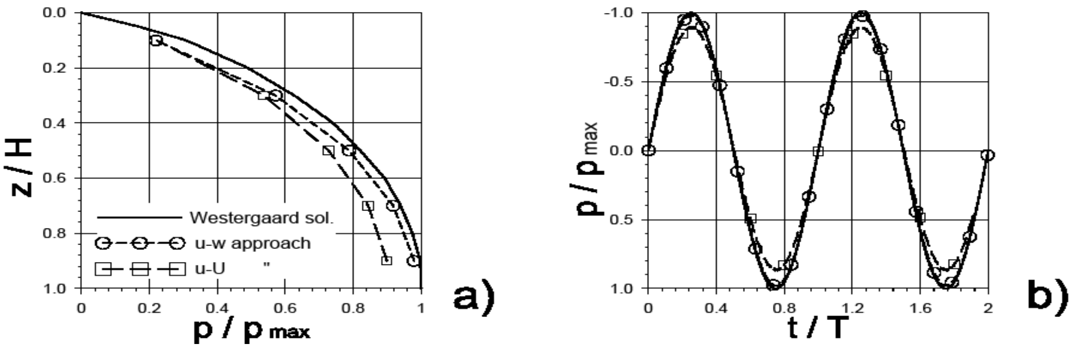

Figure 1. Maximum dynamic excess pressure distribution along the vertical wall (a) and variations with time of the dynamic excess pressure at the wall base (b): comparison of the ***u-w*** results with Westergaard solution and the results of the ***u-U*** approach.

The second example concerns a shallow excavation into saturated granular soil, supported by two flexible retaining walls (Figure 2a). The analyses were carried out by modelling the excavation and the lowering of the water table in five steps (Cividini & Gioda, 2015). Subsequently, a dynamic excitation in the horizontal direction is imposed to the bedrock which derives from the north-south component of the Tolmezzo main shock 1976 earthquake (available from the database ITACA). The excitation, lasting 15 seconds, is corrected so that at the end of it the velocity at the mesh bottom vanishes. Since the main frequency content of the considered earthquake is below 3 Hz the adopted mesh is adequate to propagate a reasonable portion of the energy input even in terms of shear waves.

Zienkiewicz & Bettess (1982) showed that, depending on the geometrical and material characteristics of the problem at hand, a fully coupled Biot dynamic analysis can be mandatory. The problem under examination falls in this category.

Figures 2 and 3 show the evolution in time of nodal displacements and of the average pore pressures obtained adopting the 'complete' ***u-w*** approach and the simplified ***u*** analysis. The displacements refer to point A located on the retaining wall (cf. Figure 2a). In Figures 2b and 2c $u_{x\ exc}$ and $u_{y\ exc}$ are, respectively, the absolute value of the horizontal and vertical displacements evaluated at the end of the excavation steps.

The diagrams in Figure 3 report the variation with time of the average pore pressure within two elements B and C. The first one is located in the deposit at the same level of the excavation bottom, while the second one is below the excavation area, close to the tip of the left wall. It can be observed that in this illustrative example the difference between the quantities evaluated with the two formulations is appreciable on the horizontal displacements and on the pore pressures while it is limited on the vertical displacements.

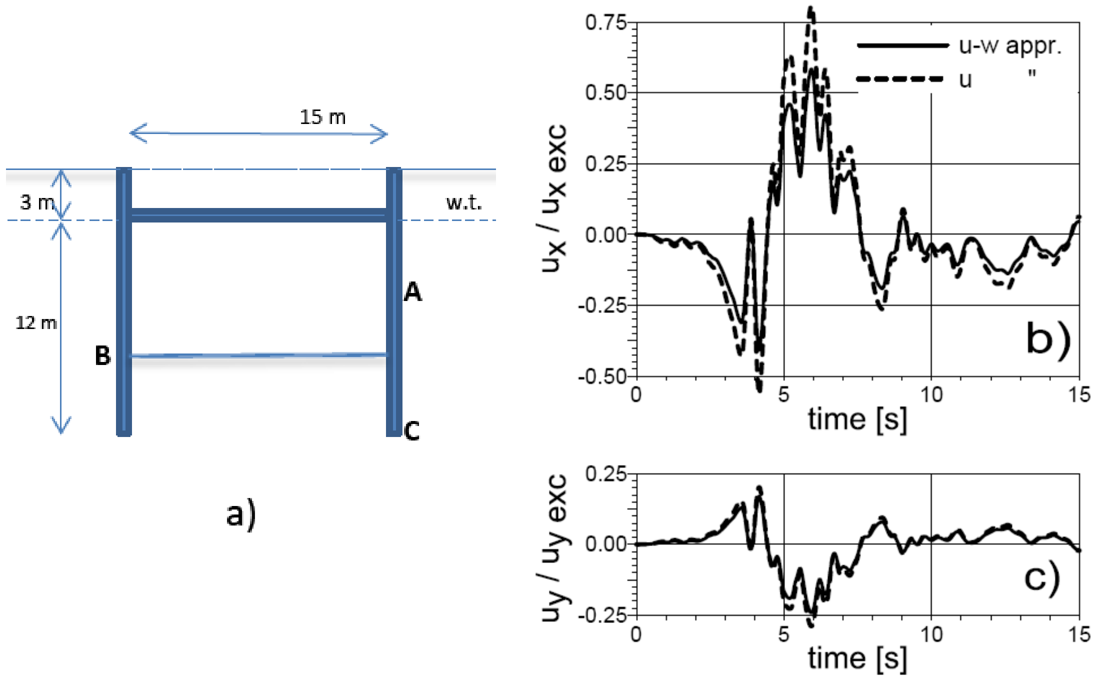

Figure 2. Scheme of the excavation supported by two flexible walls (a). Variation in time, at point A, of the horizontal displacement u_x (b) and of the vertical displacement u_y (c), calculated with the 'complete' ***u-w*** and the simplified ***u*** approaches.

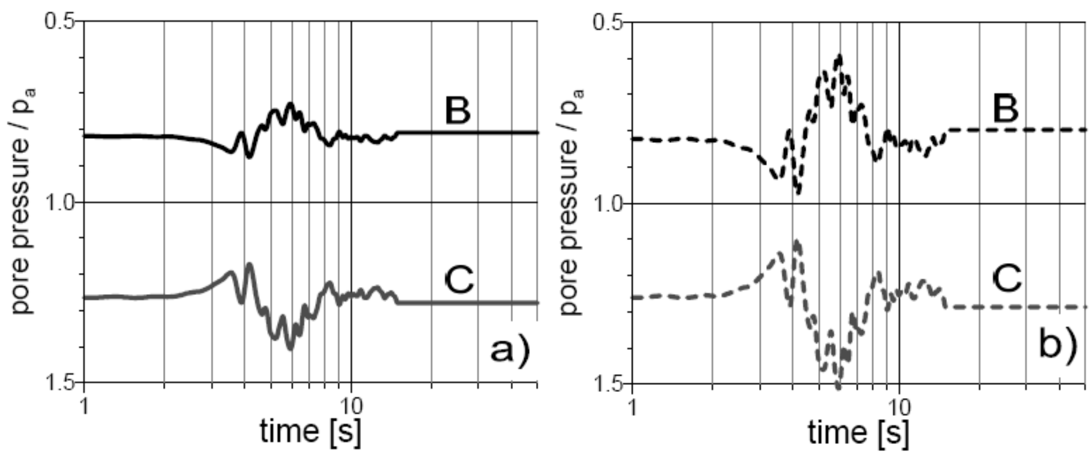

Figure 3. Average pore pressure p versus time in elements B and C: comparison between the results of 'complete' ***u-w*** (solid lines) and of simplified ***u*** (dashed lines) approaches (p_a is the atmospheric pressure value).

It is worthwhile to note that, in general, the differences between the quantities evaluated with the two formulations are appreciable not only during the earthquake motion, but also during the early stage of the post-earthquake consolidation process as shown in Cividini & Gioda (2013).

Concluding Remarks

The complete formulation of dynamic two-phase problems has been summarized introducing assumptions which seem acceptable in the geotechnical engineering context. Two finite element approaches were derived on this basis. They are referred to as the “complete” ***u-w*** and the “simplified” ***u*** formulations. The latter of them, in fact, permits reducing the number of nodal variables with respect to the first one with a consequent reduction of the computational burden.

The results obtained in the solution of a first bench mark problem, involving solely the liquid phase, showed an acceptable agreement with the corresponding analytical solution. A second text example was then solved which concerns a shallow excavation, supported by flexible retaining walls, into saturated granular soil. A discrepancy was observed in this case between the results of the two approaches. In particular, the simplified ***u*** approach provides soil displacement and pore pressure values which are somewhat greater than those obtained with the complete ***u-w*** formulation. The analysis of the causes of the possible discrepancy, and of its relevance in engineering terms, requires some further investigation, considering more severe shaking conditions and that will be part of the a subsequent study.

Acknowledgments

The CNR2012 and PRIN2008 financial support of the Ministry of University and Research of the Italian Government is gratefully acknowledged.

References

Bear, J. (1988), *Dynamics of Fluids in Porous Media*, Dover Publications, NewYork, USA.

Bird, R.B., Stewart, W.E. & Lightfoot, E.N. (2007), *Transport Phenomena*, John Wiley & Sons, NewYork, USA.

Cividini, A. & Gioda, G. (2013), Approcci ad elementi finiti per l'analisi di terreni saturi. *Atti del XV Convegno ANIDIS - L’Ingegneria Sismica in Italia*; a cura di F.Braga e C.Modena con la collaborazione di M.A.Zanini e L.Brandolin, Tema: b. Vulnerabilità e rischio sismico, Padova, 30 Giugno - 4 Luglio, Padova University Press, (Italy), ISBN 978-88-97385-59-2.

Cividini, A. & Gioda, G. (2014), Seepage flow analysis in gravity and in variable acceleration fields. *Annals of the University of Bucharest* (mathematical series), ISSN 2067-9009, **5**(2):245-258.

Cividini, A. & Gioda, G. (2015), "On the finite element formulation of dynamic two-phase coupled problems". (submitted for publication).

Cividini, A. & Pergalani, F. (1994), "Alcuni aspetti della modellazione numerica di mezzi plurifase", *Atti del Convegno CNR* - Gruppo nazionale di coordinamento per gli studi di ingegneria geotecnica. Il ruolo dei fluidi nei problemi di ingegneria geotecnica, Mondovi" (Cuneo), 6-7 settembre, 1, II/61-II/75.

Desai, C.S. (1976), "Finite element residual schemes for unconfined flow". *International Journal for Numerical Methods in Engineering*, **10**, 1415-1418.

ITACA (2015), ITalian ACcelerometric Archive, version 2.0, http://itaca.mi.ingv.it/ItacaNet.

Katona M.G. & Zienkiewicz, O.C. (1985), "A unified set of single step algorithms - Part 3: the beta-m method, a generalization of the Newmark scheme*". International Journal for Numerical Methods in Engineering*, **21**, 1345-1359.

Newmark, N.M. (1959), “A method of computation for structural dynamics”, *ASCE Journal of the Engineering Mechanics Division*, **85**(EM3), 67-94.

Westergaard, H.M. (1933), "Water pressures on dams during earthquake". *Transaction of American Society of Civil Engineers*, **98**, 418-434.

Sandhu, R.S. &Wilson, E.L. (1969), "Finite Element Analysis of Seepage in Elastic Media". *ASCE Journal of the Engineering Mechanics Division*, 95(EM3), 641-652.

Stucchi, R., Cividini, A. & Gioda, G. (2010), "A finite element approach for dynamic seepage flows", *Proc. 7th European Conference on Numerical Methods in Geotechnical Engineering*, June 2-4, Trondheim (Norway), ISBN 978-0-415-59239-0, pp.411-416.

Zaman, M., Gioda, G. & Booker, J. (eds.) (2000), *Modeling in Geomechanics,* John Wiley & Sons, Chichester, UK.

Zienkiewicz, O.C. & Bettess, P. (1982), "Soil and saturated media under transient, dynamic conditions; general formulation and the validity of various simplifying assumption". In *Soil Mechanics - Transient and Cyclic Loads: Constitutive Relations and Numerical Treatments* (Pande, G.N., Zienkiewicz, O.C., Eds.), Wiley Series in Numerical Methods in Engineering.

Zienkiewicz, O.C. & Shiomi, T. (1984), "Dynamic behaviour of saturated porous media; the generalized Biot formulation and its numerical solution". *International Journal for Numerical and Analytical Methods in Geomechanics*, **8**, 71-96.

Zienkiewicz, O.C., Chan, A.H.C., Pastor, M., Schrefler, B.A. & Shiomi, T. (1999), *Computational Geomechanics*, John Wiley & Sons, Chichester, UK.