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Empirical Excess Pore Pressure Dissipation Model for Liquefiable Sands

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ABSTRACT

In the early phase of excess pore pressure dissipation, the excess pore pressure in a liquefied homogeneous soil stratum reduces most significantly at the base, leading to a curvature at the lower part of the initial isochrone while the upper half remains fairly unchanged. In the later phase, the curvature leaves the initial isochrone completely and progresses until excess pore pressure is fully dissipated. In this paper, a simplified exponential equation is introduced to capture the excess pore pressure dissipation over time. This equation is applied to several dynamic centrifuge tests and shown to depict a suitable fit to the dissipation time history over a wide range of hydraulic conductivity of sands.

Introduction

Soil liquefaction has been studied extensively over the past several decades since its field observation following the 1964 Niigata Earthquake. However, it still poses serious threats to infrastructure in earthquake prone areas to date, where liquefiable soil layers are present. This is evident in recent earthquakes such as the Christchurch Earthquake and the Great East Japan Earthquake in 2011. Numerous researchers have taken notice of the loss of shear strength due to generation of excess pore pressure during earthquake shaking, however not many have investigated the impact of dissipation of excess pore pressure as thoroughly. Severe ground displacement/settlement can occur during the dissipation of excess pore water pressure. This can lead to tilting of buildings, sliding/lowering of dam crest, lateral spreading of bridge abutment wall, etc. In view of fewer studies on pore pressure dissipation, a detailed investigation was conducted with the objective to depict the excess pore pressure dissipation time history of liquefied soil under vertical drainage.

Earthquake Induced Liquefaction

Soil liquefaction is a physical process occurring frequently in loose saturated sandy deposits. When the soil is subjected to shear during an earthquake ground motion, the loose soil grains tend to compact into a denser state, leading to the collapse of pore voids within the soil-water matrix. However, given the rapid cyclic shear loading, the fluid water in these collapsing pore voids is unable to escape fast enough, thus leading to an undrained condition. Rather than a volumetric contraction, the consequence is an increase in pore water pressure which reduces the frictional contact surface between the soil grains; hence a reduction in effective stress. At this stage, the shear strength of the soil is reduced dramatically while the soil grains initially held

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together are separated and suspended in water. As a result, the soil behaves in a free-flowing condition and deforms effortlessly with a slight pressure. Based on the following effective stress principle, the effect of soil liquefaction due to the increase in pore water pressure *u*, can be understood by the loss of vertical effective stress σ'_v , as shown below in Eq. 1:

$$\sigma'_v = \sigma_v - u \tag{1}$$

After earthquake shaking has ceased, the soil particles settle in a sedimentation process where particles regain contact and begin taking load. Excess pore pressure dissipates as commonly expressed as the classic 1-D consolidation equation in Eq. 2:

$$\frac{\partial \overline{u}}{\partial t} = C_v \cdot \frac{\partial^2 \overline{u}}{\partial z^2} \tag{2}$$

where $\bar{u}$ refers to the excess pore pressure, t is time, z is the depth of the soil and C_v is the coefficient of consolidation in the vertical direction. C_v is directly proportional to the soil's permeability k, and inversely proportional to the compressibility index m_v and the fluid unit weight γ_w as shown in Eq. 3:

$$C_v = \frac{k}{m_v \cdot \gamma_w} \tag{3}$$

The hydraulic conductivity of the soil K (i.e. measure of how easily the pore fluid moves through the soil) is dependent on the permeability of the soil k and density of pore fluid ρ_w. μ and g are the dynamic viscosity of fluid and acceleration due to gravity respectively.

$$K = k\frac{\rho_w g}{\mu} \tag{4}$$

Excess Pore Pressure Dissipation Models

Several excess pore pressure dissipation models have been developed based on results of shaking table and centrifuge tests. In 1-g shaking table tests, Florin and Ivanov (1961) observed that the curves representing maximum excess hydraulic pressure versus soil depth resembled a trapezoidal profile. The progression of liquefaction commenced from the surface to greater depth of the soil. After shaking, consolidation took place and excess pore pressure decreased with the boundary of the consolidated soil moving upwards to the surface. Dissipation of excess pore pressure was more rapid in more permeable coarse-grained soils as compared to fine-grained ones.

Scott (1986) suggested that the trapezoidal profile from Florin and Ivanov (1961) is applicable only for rigid soil grains which do not compress significantly with overburden stress as the solidification layer increases. The settled solid layer is subjected to compression as the solidified thickness increases and pore fluid expelled from the soil. The excess pore pressure at any particular depth in the soil layer is equivalent to the sum of the pressure of the overlying buoyant unit weight of the soil γ' above the solidified layer plus the consolidation pore pressure. The velocity of bottom-up solidification is assumed to be constant. Using two centrifuge tests from Lambe (1981) using the beam centrifuge at the University of Cambridge, the experimental excess pore pressure dissipation data was back-analysed to obtain the soil properties. This approach was however based on rough approximation with the predicted dissipation time history usually underestimating the experimental data, especially at greater depths where the model is unable to fit the curvature of the dissipation time history as shown in his paper (Scott, 1986).

Previous research at the University of Cambridge by Brennan and Madabhushi (2011) proposed a Fourier series to fit the curvature of dissipating excess pore pressure following a full liquefaction event in centrifuge tests as shown in Eq. 5.

$$\bar{u}(z,t) = \sum_{n} \frac{8 \cdot \gamma' \cdot H}{n^2 \cdot \pi^2} \cdot \sin\left(\frac{n \cdot \pi \cdot z}{2 \cdot H}\right) \cdot \sin\left(\frac{n \cdot \pi}{2}\right) \cdot \exp\left(-C_v \cdot \left(\frac{n \cdot \pi}{2 \cdot H}\right)^2 \cdot t\right) \qquad (5)$$

where H and z are the height of the entire soil stratum and the specific soil depth in-study respectively. If experimental data is present, the C_v parameter of the soil can be obtained. The above Fourier series can be readily fitted to experimental data and automatically satisfies the boundary conditions of $\bar{u} = 0$ at $z=0$ and $\partial \bar{u} / \partial z = 0$ at $z=H$. The model however involves a number of terms which may over-fit curves onto noisy data points.

Kim et al. (2009) also developed an empirical dissipation model as expressed in Eqs. 6 and 7. The model was developed by combining the non-linear model of the solidification velocity with Scott's (1986) theory.

$$H = \frac{t}{a + b \cdot t} + c \cdot t \qquad (6)$$

$$u_G(z, t+dt) = u_G(z,t) + \beta(u_G(z+dz,t) + u_G(z-dz,t) - 2u_G(z,t)) + \gamma' \cdot dH \qquad (7)$$

where u_G is the excess pore pressure, H is the thickness of solidified layer, a and b are parameters used to simulate initial solidification velocity near the bottom, and c is the parameter used to simulate final solidification velocity near the surface of the soil deposit. β is a consolidation dependent parameter ($= c_v \cdot dt / dz^2$). The advantage of this method is that it corrects Scott's (1986) assumption that the solidification velocity is constant. The model however introduces 3 additional parameters (a, b and c) which are dependent on the D_{30} grain size and relative density (D_r) of the soil.

Dynamic Centrifuge Testing

In order to assess excess pore pressure dissipation of liquefied sands in great detail, data from a wide range of hydraulic conductivity of sands have to be collected. A deep column of homogeneous sand deposit is needed to measure the dissipation of excess pore pressure; however this is not feasible for 1-g shaking tests. Centrifuge testing was therefore conducted to achieve depth of saturated soil exceeding 10m.

Soil is a highly non-linear material. It is therefore essential to replicate identical stress and strain conditions in laboratory tests as in the prototype scale. Geotechnical centrifuge modelling achieves these conditions with the use of high centrifugal acceleration to scale up the model. A scaled model is made to correspond with the prototype at the pre-determined centrifuge g-level. As a result, a 1:N scale model experiences the same stress and strain conditions as the prototype when subjected to a centrifugal acceleration of N times of the earth's gravity (Schofield, 1981). It is recognized in physical modeling that there is a disparity between the scaling law for time for diffusion processes (such as consolidation given by $1/N^2$) and that for dynamic events (given by $1/N$). This disparity is resolved by using a pore fluid of viscosity N times greater than water (normal pore fluid). By increasing the viscosity of the pore fluid, both the rate of excess pore pressure generation (due to earthquake loading) and the rate of dissipation (due to soil consolidation) are matched.

The model container used in the centrifuge tests at Cambridge University was the Equivalent Shear Beam (ESB) Box. The design and performance compliance of the ESB Box was described by Zeng and Schofield (1996) and Teymur (2002). An automatic sand pourer was used to prepare the loose sand model at 45% relative density based on design charts produced by Chian *et al.* (2010). The sand properties are described in Table 1.

Table 1. Properties of Hostun and Fraction E sands

Properties	**Hostun**	**Fraction E**
ϕ_{crit}	33^o	32^o
D_{10}	0.209 mm	0.0993 mm
D_{50}	0.335 mm	0.195 mm
e_{min}	0.555	0.613
e_{max}	1.01	1.014
G_s	2.65	2.65

After sand pouring was completed, the sand models were saturated with high viscous methyl cellulose fluid prepared at the desired viscosity (cSt) equivalent to the centrifuge g-level in the first instance, so as to satisfy scaling laws. In order to produce wider range of hydraulic conductivity of the same sand, the viscosity of the pore fluid was altered. Table 2 shows the range of hydraulic conductivity of the sand model with change in sand type and/or viscosity of pore fluid. Data from published literatures are included in the table for subsequent validation of analysis.

Fig 1 shows the excess pore pressure generation and dissipation in Tests 1 and 4. As observed in the figure, tests with soil of higher hydraulic conductivity (i.e. Test 4) generated lower peak excess pore pressures during shaking. This is because high excess pore pressure was unable to be retained for long given the high hydraulic conductivity of the soil. Similarly, Test 4 dissipates excess pore pressure at a much higher rate for the same reason. Observations from Fig. 2 also showed that excess pore pressure dissipation commences from the linear line (value equivalent to the initial effective stress), representing full liquefaction of the sand. In the early phase of the dissipation process, the excess pore pressure reduces most significantly at the base, thereby leading to a curvature at the lower part of the initial isochrone while the upper half remains fairly unchanged. This infers that high excess pore pressure was maintained for a relatively long period of time at shallow depth of the soil. As the pore fluid migrates from the base of the liquefying soil layer to the soil surface as the sole drainage path, it retains the high pore pressures at shallow depths of the soil. The high pore pressure is maintained until the underlying excess pore pressure approaches the value at the shallow depth of the soil. Thereafter excess pore pressure at the shallow depths decreases and leaves the initial isochrones completely as shown in Fig. 2. Reduction in excess pore pressure progresses towards the y-axis until the excess pore pressure is fully dissipated, akin to the excess pore pressure at greater depths

Table 2. Types of sand and their hydraulic conductivities in centrifuge tests

Test	**Sand Type**	**Pore fluid viscosity / centrifuge g-level (cSt/g)**	**Hydraulic conductivity ($\times 10^{-4}$ m/s)**	**Reference**
1	Fraction E	1	0.99	This study
2	Hostun	1	4.37	This study
3	Hostun	1/3	13.1	This study
4	Hostun	1/9	39.3	This study
Brennan-E	Fraction E	1	0.99	Brennan, 2004
Kim-J	J-Sand	1	18.1	Kim et al, 2009
Tobita-7	Silica Sand No.7	1	0.81	Tobita and Iai, 2011

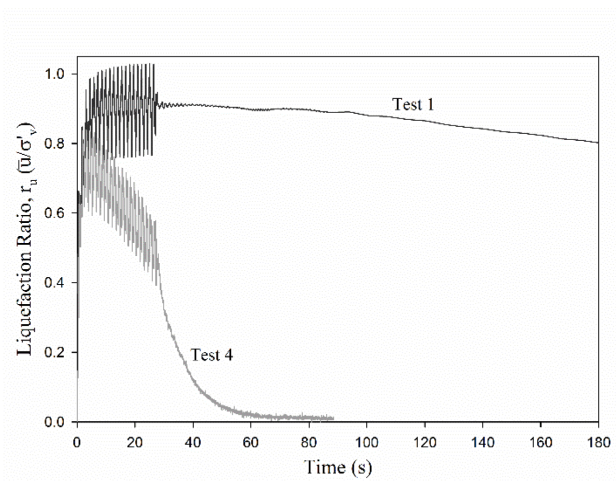

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Fig. 1 Excess pore pressure at 13m soil depth, Tests 1 and 4

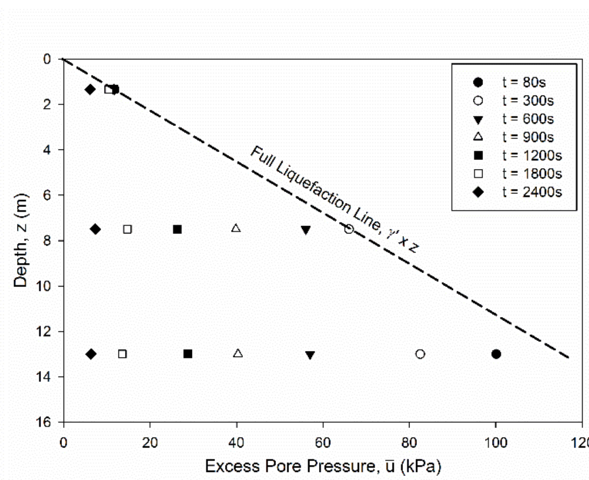

Fig. 2 Excess pore pressure isochrones Test 1

Motivation for an Improved Dissipation Model

As briefly discussed earlier, the dissipation of excess pore pressure is typically idealised as a self-weight consolidation of a hydraulic fill. When soil particles are not initially in contact (i.e. effective stress is zero), the total weight of the soil is carried by the pore water pressure. These soil particles settle through the water until they come in contact with each other. At this time, the soil gradually consolidates and carries its own weight through the interparticle contact forces as excess pore pressure dissipates. The soil layer is compressed and excess pore pressure drains out of it. Typically, consolidation is analysed approximately as parabolic-shaped isochrones in those cases. However, due to the nature of the parabola, the linear full liquefaction line as shown in Fig. 2 cannot be produced at the start of the dissipation process (i.e. t=0s). The Fourier series proposed by Brennan and Madabhushi (2011) as expressed in Eq. 5 may be capable of providing a solution for the above concern, but the equation is fairly complex. An alternative simpler mathematical equation is hence desirable. This equation should be time-dependent and without too many variables.

Proposed Dissipation Model

The excess pore pressure time history can be expressed as two functions, one involving depth and the other with time as shown in Eq. 8:

$$\bar{u}(x,t) = (c_1 \sin \alpha x + c_2 \cos \alpha x) \cdot e^{(-\alpha^2 C_v t)} \tag{8}$$

where c_1, c_2 and α are constants, x is the ratio of a specific soil depth and the height of the entire soil stratum. The equation has to satisfy the following two boundary conditions:

$$\bar{u}(0,t) = 0 \tag{9a}$$
$$\bar{u}(x,0) = x \cdot u_i \tag{9b}$$

where u_i is the initial effective overburden stress (= $\gamma' z$). Eq. 8 is simplified to:

$$\bar{u}(t) = e^{(-\alpha^2 C_v t)} = e^{(-At)} \tag{10}$$

where *A* is a constant. It can be observed that as a result of the boundary conditions in Eq. 9, the simplified equation in Eq. 10 no longer has a depth variable. This implies that the excess pore pressure dissipation time history follows an exponential function and is independent of depth. In addition, the use of exponential decay function for similar deposition process is commonly observed in natural sedimentary basins as well (Athy, 1930). Furthermore, the independence of the excess pore pressure dissipation with depth is analogous to the earlier hypothesis that excess pore pressure at shallow depth can only dissipate after the underlying excess pore pressure approaches the same value. A trapezoidal excess pore pressure versus depth profile is therefore confirmed as observed in Florin and Ivanov (1961) experiments. Figs. 3 and 4 show the excess pore pressure measurements of two centrifuge tests overlaid with the exponential model of Eq. 10.

As observed in Figs. 3 and 4, it is apparent that despite fitting the excess pore pressure measurements near the base of the centrifuge soil model, the shallower depths tend to deviate from the proposed exponential model. This is due to progressive settlement of the soil as dissipation takes place. Dissipation of excess pore pressure leads to reduction in pore space, hence the settlement of the soil. The degree of compaction differs with depth of the soil as overburden stress at greater depth of the soil would lead to larger settlement of that layer when soil grains regain contact, hence leading to the non-linear “solidification velocity front” postulated by Scott (1986). Due to differences in degree of compaction, excess pore pressure measurements would deviate from the trapezoidal excess pore pressure versus depth profile more significantly for shallower soil depths. However, the trapezoidal profile can be easily adjusted to reflect the non-linear rate of settlement of the soil with time as the amount of settlement can be computed from the area between excess pore pressure isochrones at different time frames. Upon adjusting for the compaction, excess pore pressure isochrones at shallow depths would be shifted slightly towards the left as the height of the soil stratum would have decreased due to settlement. However, given that ground settlement in most cases is less than 10% of the entire depth of liquefiable stratum, the effect of such adjustment is often negligible.

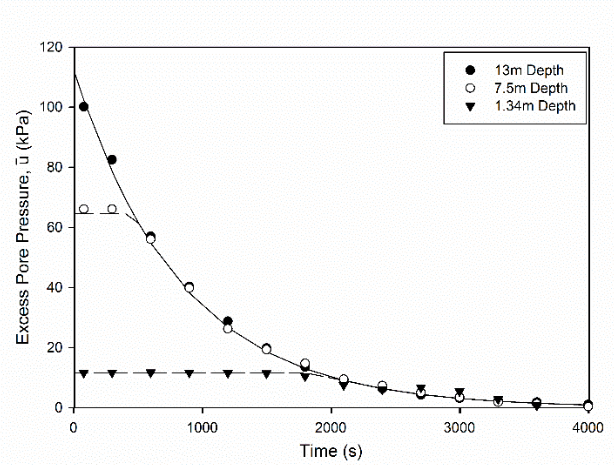

Fig. 3 Excess pore pressure dissipation overlaid with exponential function, Test 1

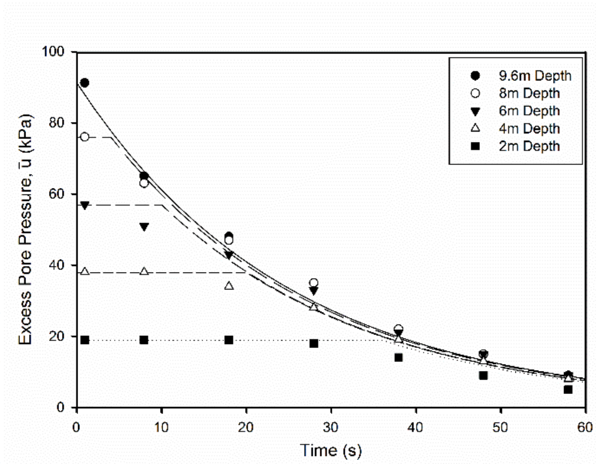

Fig. 4 Settlement estimated from area under excess pore pressure isochrones, Test Kim-J

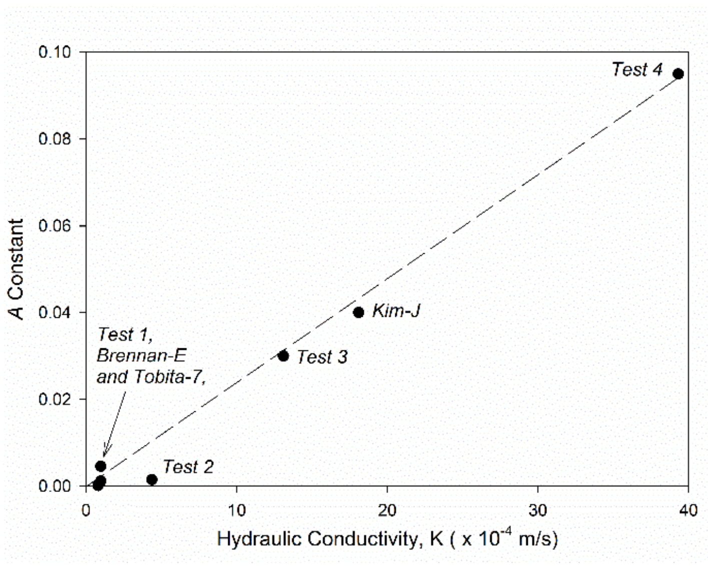

Fig. 5. Relationship of constant *A* with hydraulic conductivity of sand

A fitting analysis was conducted to obtain an estimate of the constant, *A* in Eq. 10 for all the centrifuge tests. Fig. 5 illustrates the *A* constant plotted with respect to the hydraulic conductivity of the sand models. Data from other published literatures are also analysed and plotted as validation. As shown in the figure, it is evident that the constant *A* can be correlated linearly to hydraulic conductivity. This is logical as a higher sand permeability would allow rapid dissipation of excess pore pressure and hence a steeper exponential function. In addition, the constant A has a coefficient of consolidation (C_v) component which is influenced by the hydraulic conductivity of the soil as shown in Eqs. 3 and 4. As high hydraulic conductivity soils in liquefaction tests were rarely carried out, the available range of hydraulic conductivity of sands in published database is limited for more rigorous validation. Nevertheless, the strong linearity relationship between the constant A and the hydraulic conductivity of the sand with the available test data presented in this paper is encouraging and may form a basis for quick prediction of excess pore pressure dissipation for different permeability sands.

Conclusion

A simplified model consisting of an exponential function with a fitting constant is proposed in this paper. The model was validated with several centrifuge tests over a wide range of hydraulic conductivity of sands conducted by the author and data from published papers. It was found that the fitting constant at the power of the exponential function is linearly proportion to the hydraulic conductivity of the sand deposit and may form a convenient basis for quick prediction of excess pore pressure dissipation for different permeability sands.

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