

# An Assessment of Pseudo-Static Analysis Methods for Practical Seismic Design of Flexible Retaining Structures

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## ABSTRACT

The seismic analysis of flexible retaining structures may represent a very challenging task for soil engineers if all the physical aspects were to be considered in a rigorous manner. However, in low seismic zones or limited to preliminary design stages, simplified analysis methods are still widely used. In this respect, this paper aims at addressing some common issues of seismic analysis with the non linear spring method, which is still one of the most popular methods in the practice. An improvement to include seismic effects within a pseudo-static framework is presented. Some examples and comparison with other design methods are discussed.

## Introduction

Flexible retaining wall design currently requires a careful assessment of many geotechnical and structural aspects, as well as of economical and even legal issues. In current practice, numerical analyses are routinely used, based on finite element or finite difference codes. Even so, by practicing engineers, traditional approaches are still in use, including closed form formulas, slope stability methods, and the so called non-linear spring method, at least in the early design stages.

In seismic zones, pseudo-static analysis is the most common method in the design of retaining walls. In the framework of the Performance Based Design Method (PBDM), traditional pseudo-static calculation methods are receiving a renewed attention even by research studies from academia. In fact PBDM requires the calculation of an ultimate wall capacity, which is usually available, for many practical cases, by simple limit equilibrium methods, such as the Blum method (Blum (1931)). Moreover, pseudo-static analysis suffers from an intrinsic limitation in predicting real wall deformations (Kontoe et al. (2013)): this aspect often discourages the selection of unnecessarily complex modelling tools, in a pseudo-static analysis, since the accuracy increase provided by such methods is, in most cases, unreliable. Hence the assessment and—possibly—the improvement of traditional retaining wall analysis methods for seismic calculation is still a valuable topic for applied research. In this respect, we'll first briefly review some aspects of the non-linear spring method implemented in our commercial code PARATIE PLUS™ 2014 (PPLUS in the following) (Ce.A.S. (2014)), including an option (Becci & Carni (2014)) to automatically select seismic actions depending on wall behaviour. The results and limitations of this proposal will be discussed through examples, considering some tentative preliminary yet practical conclusions in the light of PBDM.

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## The Non-Linear Spring Method in PARATIE PLUS 2014

The non-linear spring method is an engineering approach to assess the lateral behaviour of a flexible retaining wall. Such method is normally implemented in a finite element program, in which the wall is modeled by beam elements and the interaction with soil at each face of the wall by special non-linear spring elements, which, in PPLUS, are lumped springs at nodes.

### *Static behaviour*

The constitutive model of such springs is outlined in Figure 1: starting from a self-balanced distribution of at rest lateral pressures  $\sigma'_{h0}=K_0 \cdot \sigma'_v$ , subsequent stages are studied in which excavations or insertion of props is modelled. Based on lateral wall deformations  $\Delta$ , soil elements react as shown, within active and passive yield limits related to current effective vertical stress  $\sigma'_v$ .

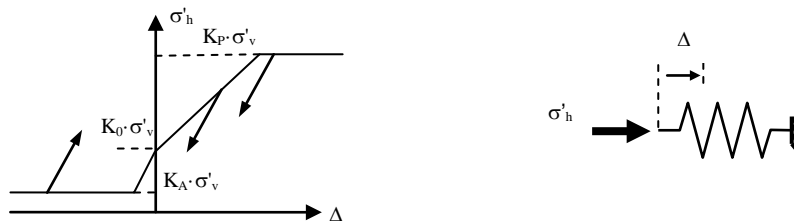


Figure 1: Constitutive law for a non-linear spring (cohesionless soil).

Normally, active thrust coefficients  $K_A$  are determined according to classical Coulomb equations whereas passive values  $K_P$  by Caquot et al. (1973), accounting for wall-soil friction  $\delta$  and dredge line slope. Spring stiffnesses are related to the secant elastic soil moduli and to the excavation geometry (wall height and toe depth) by means of simple equations (Becci & Nova (1985), Becci & Carni (2014)). At each analysis stage, the solution is obtained by Newton-Raphson iterations. The straight-forward model definition as well as the simplicity in soil parameters selection makes such kind of procedure a quite attractive option in several design situations.

### *Pseudo-static seismic analysis - traditional method*

A traditional approach for pseudo-static non-linear spring analysis can be summarized in the following steps (Figure 2(a)):

1. the excavation process is modeled to compute static lateral effective stresses  $\sigma'_{h,s}(z)$ ;
2. some seismic thrust increments  $\Delta p_E$  at the driving side of the wall are added, as external loads, to the soil pressures at the driving side;
3. at the resisting side,  $K_P$  is modified accounting for seismic reduction.

$\Delta p_E$  should be selected in the light of the presumed wall behaviour. If the soil at the driving side is expected to yield during seismic event, for ordinary dry granular backfills, the Mononobe & Okabe (M-O) increment to static active pressures may be applied as a constant pressure

distribution given by Equation 1 (Seed & Whitman (1970)).

$$\Delta p_E = \Delta p_{E,M-O} \approx 0.375 \cdot \beta \cdot \left( \frac{a}{g} \right) \cdot \gamma \cdot H \quad (1)$$

where  $H$  is the wall height,  $\gamma$  the average backfill unit weight,  $(a/g)$  the normalized seismic acceleration and  $\beta < 1$  is a PBDM reduction factor accounting for wall ductility, which, according to Italian Code (NTC 2008) may be computed by Equation 2:

$$\beta = 0.12658 \cdot \ln \left( \frac{1.80}{u_s [m]} \right) \quad u_s = \text{accepted wall deformation} \leq 0.5\% H \quad (2)$$

When the wall is expected to behave *elastically*, Equation 1 is not appropriate and, for example, the popular Wood (1973) equation may be considered:

$$\Delta p_E = \Delta p_{E,WOOD} = \left( \frac{a}{g} \right) \cdot \gamma \cdot H \quad (3)$$

Comparing Equation 3, in which of course  $\beta$  must not be included, with Equation 1, it is apparent that the *a priori* assumptions on the expected wall deformations greatly affect the results.

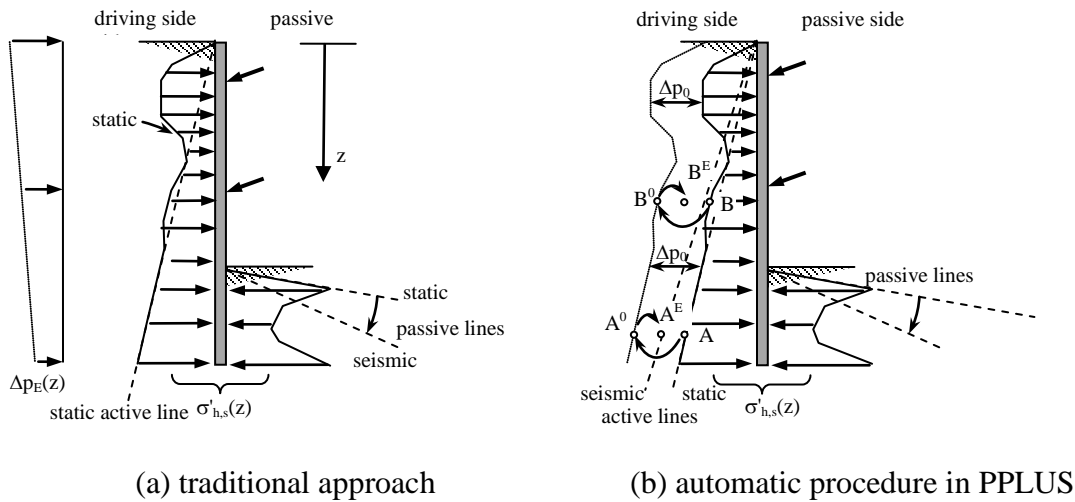


Figure 2: Seismic modifications to static pressures.

### ***Pseudo-static seismic analysis - PPLUS procedure***

The following procedure is proposed aiming at linking seismic thrust increments to actual wall stiffness (see Becci & Carni (2014) for further details). In uphill soil regions (e.g. points A or B in Figure 2(b)), for a while, incremental deformations may be assumed to be negligible and the seismic thrust increment may be estimated by a rigid (elastic) approach (Equation 3): static driving pressures temporarily move to the leftmost dotted line in Figure 2(b) (i.e.  $A \rightarrow A^0$  and

$B \rightarrow B^0$ ). Due to such increase, equilibrium iterations are required which may increase wall deflections and reduce the final thrusts (i.e.  $A^0 \rightarrow A^E$  and  $B^0 \rightarrow B^E$ ).

Such behaviour is attained by means of the following two simple operations.

1. In iteration 0, in any uphill soil element only, the effective lateral pressure is increased by rigid increment  $\Delta p_0$  (e.g. by Equation 3), at zero strain increment.
2. During further iterations, strain increments are allowed and, at the same time, yield limits are updated to the seismic values  $K_{A,E}$  (using M-O equations) and  $K_{P,E}$  (determined, by PPLUS, according to Lancellotta (2007)).

Computed seismic pressures fall between a minimum (active) and a maximum (e.g. Wood) distribution, depending on wall flexibility, without requiring preliminary assumptions. On the other hand, such proposal suffers from some severe approximations: for example, the decrease rate from Wood to active conditions is governed by the same stiffness considered in static calculations; moreover the position of the overall active seismic thrust is essentially the same as the static active thrust.

### Examples

In the technical literature, experimental studies are mostly limited to cantilevered or singly anchored walls. While such studies (e.g. Sitar et al. (2012)) offer precious general insights, in order to evaluate the effectiveness of practical design tools, some benchmarks which are as similar as possible to the current practice would be very valuable. For relevant excavation depths supported by multi-propped flexible walls, most of the available seismic studies are based on numerical analyses conducted at the design stage of the problem at hand (e.g. O'Riordan & Almufti (2014)). In the light of the considerations above, also the examples below cannot but stem from numerical rather than from experimental studies.

#### *Example 1: singly-propped trench (Callisto et al. (2009))*

A numerical study (Callisto et al. (2009)) of a 4 m deep trench supported by r.c. walls is considered, in Figure 3. Original soil parameters have been converted in the simpler PPLUS data, using the soil properties shown.

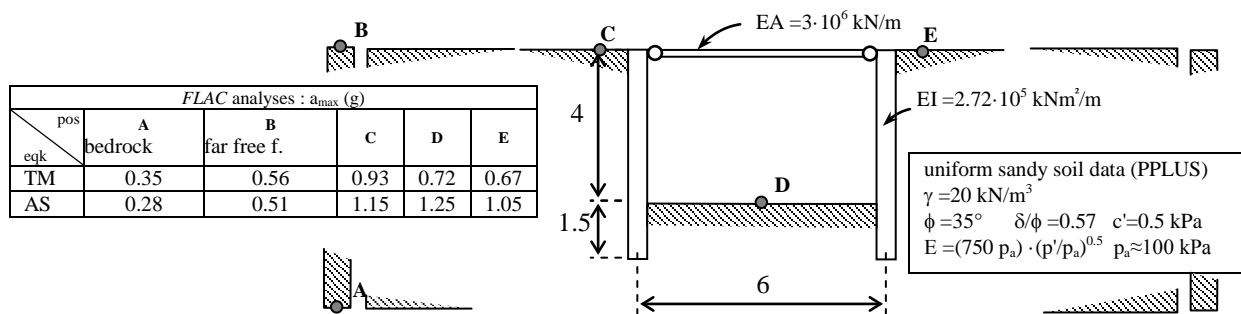


Figure 3: Example 1: A trench problem - Callisto et al. (2009).

Cited results have been obtained by means of a FLAC (Itasca (2005)) 2D dynamic analyses with two different natural recorded motions, namely TM (Tolmezzo) and AS (Assisi). Maximum computed accelerations are also summarized in Figure 3. As for PPLUS analysis, a reference value ( $\mathbf{a}/g$ ) $\approx 0.56$  will be assumed. Since in a pseudo-static approach such maximum acceleration is not compatible with the available resistances, in the light of the PBDM the following procedure is adopted (Figure 4):

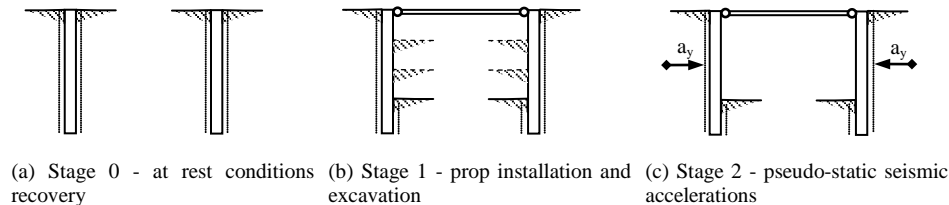


Figure 4: Example 1 - PPLUS analysis.

1. At rest conditions are restored (Stage 0 in Figure 4) as required by PPLUS in order to properly initialize the constitutive model of the non linear springs (shown as dotted lines in Figure 4).
2. Prop is installed and Excavation is modeled (Stage 1) by removing the springs above the dredge line and updating vertical stress accordingly. In this respect, it should be noted that the non linear springs at the excavation side cannot model the interaction between the opposite walls through the soil mass.
3. By means of the proposed pseudo-static procedure, maximum acceleration  $a_y$  is computed by progressively reducing  $\mathbf{a}$  until equilibrium is attained (Stage 2):  $a_y$  is therefore the maximum allowable pseudo-static acceleration compatible with the available wall static capacity. It's worth noting that, due to the simplified interaction scheme provided by PPLUS, just opposite accelerations on the facing walls can be modelled with some realism.

In determining  $a_y$ , three options have been investigated, namely:

- a)  $\Delta p_0$  at iteration 0 is computed based on  $\mathbf{a}=0.56$  g in Eqn. 3, whereas reduced  $a_y$  is used just to compute  $K_{A,E}$  and  $K_{P,E}$ . This approach corresponds with the most rigorous method in the light of the discussion above: this analysis will be referred to as "**100% Wood**"
- b)  $\Delta p_0$  from Eqn. 3 is computed using the reduced acceleration  $a_y$ : such analysis will be referred to as "**57% Wood**"
- c)  $\Delta p_0$  is not applied thus reproducing a traditional approach ("**No Wood**" analysis)

For all such cases, the same maximum acceleration  $a_y=0.32g$  is obtained, corresponding with  $\beta=0.5714$ . As expected,  $\Delta p_0$  does not affect the ultimate wall capacity.

### *Result discussion*

In Figure 5, main original and PPLUS results are compared. The predicted pseudo-static results very well agree with FLAC results. Bending moments are not affected by  $\Delta p_0$  and lay in between

maximum and residual AS earthquake results. Prop force, on the contrary, is affected by  $\Delta p_0$ : using the **100%** or **57%** assumption, PPLUS value very well matches FLAC value for AS input. Wall displacements, not shown in details here, are very different: with FLAC, inward wall toe movement larger than 20 mm is predicted, whereas in PPLUS only 11 mm is computed for the 100% Wood assumption. Such discrepancy, however, is consistent with the great differences between the two methods, since only FLAC movements incorporate a relevant part of plastic deformations developing when the static resistance is exceeded by the inertia forces. Such finding is general, making the direct comparison between dynamic and pseudo-static displacements quite deceptive.

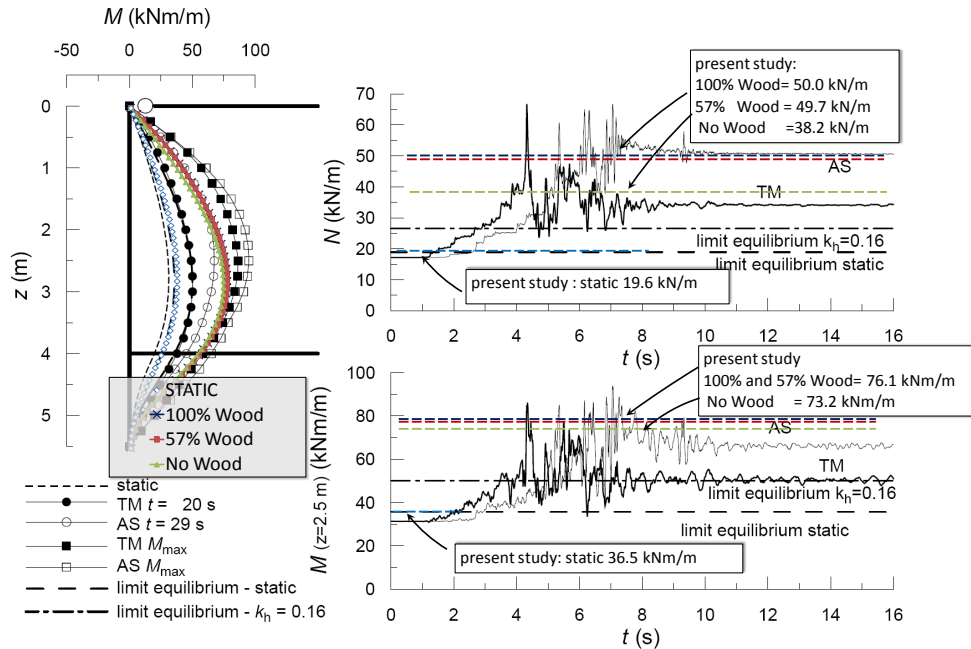


Figure 5: Example 1 - result summary.

Keeping in mind that seismic deformations should be assessed by much more complex methods, as recommended by Kontoe et al (2013), a very preliminary estimate of total deflection may be attempted as well: once  $a_y$  or  $\beta$  is available, one may use Eqn. 2 to estimate the irreversible part  $u_s$  of the deformation to be added to the pseudo-static deformed shape. Conversely, one may enter Equation 2, with a known  $u_s$  value and compute  $\beta$ . In this case, with  $a_y=0.32$ , we would obtain  $\beta=0.57$  and  $u_s \approx 2$  cm. Adding  $u_s$  to PPLUS toe movement, we fairly approach FLAC results.

### Example 2: An anchored wall in Berlin sand

A 21.4 m high r.c. bulkhead in the city of Berlin (Germany) was instrumented during underwater excavation stages; later a 2D finite element study was performed by Nikolinakou et al. (2011) employing the advanced MIT-S1 constitutive model. Some aspect of this problem can be also analyzed with PPLUS: starting from the reproduction of the documented results, a seismic analysis is conducted, to assess the response of the proposed method. General data with assumed

stratigraphy and soil properties considered in PPLUS is shown in Figure 6(a). It should be noted that PPLUS elastic moduli have been assessed based on the soil densities (void ratios  $e$ , shown in Figure 6(a)) reported by Nikolinakou et al. (2011). Measures and numerical results reported by those Authors have been reanalyzed by PPLUS, obtaining the results in Fig. 6(b). Due to the excellent comparison with the benchmark, this PPLUS model can be considered a valuable starting point for seismic sensitivity calculations. In the final layout, before dewatering, the base of the excavation was sealed by a 1.5 m thick concrete slab stabilized by tension piles: this stage has been also modelled by PPLUS. Finally two scenarios (Fig. 6(c)) have been investigated, namely **A**) a **0.1 g** earthquake occurring with ground anchor and bottom slab only acting as supports, and **B**) two rigid supports at wall top and at mid span.

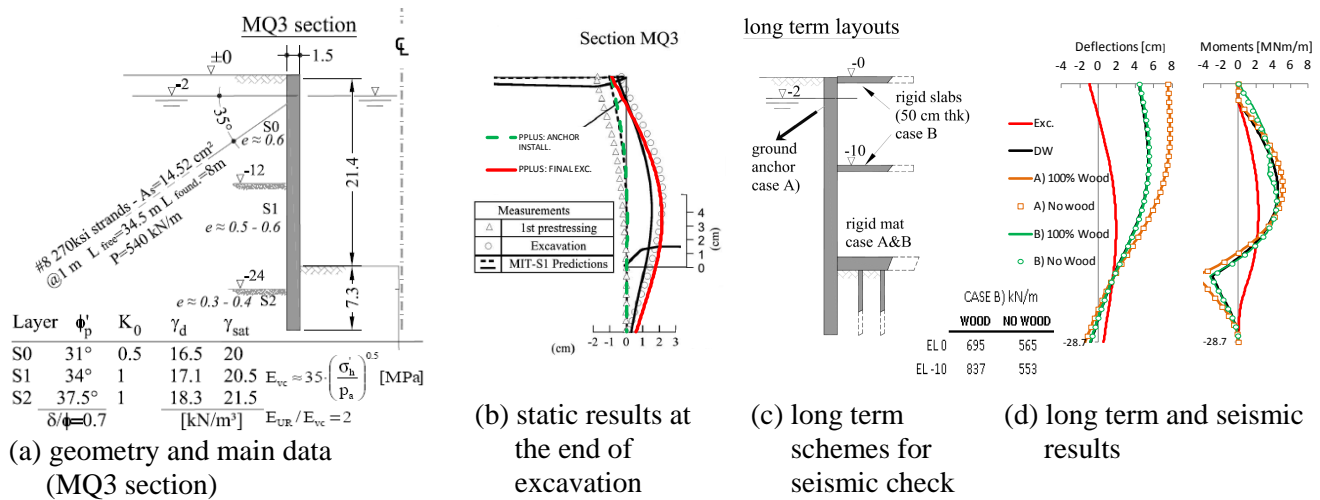


Figure 6: Anchored wall in Berlin sand.

For both cases, **100%** or **No Wood**  $\Delta p_0$  options, as in Example 1, have been assessed. Submerged soil correction and hydrodynamic water pressures have been included, as recommended by Eurocode 8 Part 5 (EN 1998-5), assuming dynamically pervious conditions.

### Result discussion

In Fig. 6(d) deflections and bending moments are depicted for initial stages (**Exc**=excavation=same as Fig. 6(b); **DW**=Dewatering conditions) and for seismic sets **A**) and **B**) (both with % 100 or without  $\Delta p_0$  pressure). In this example, Wood increments at iteration 0 do not produce significant bending moments or deflection increase with respect to ordinary M-O approach, even for rigid support (scheme B). This behaviour is likely related to the very large wall flexibility due to large spans between supports. Also very relevant static bending moments at dewatering stage concur in making the seismic increments quite negligible. For case B), however, intermediate support forces increase significantly, if initial Wood pressures are included. Again, the PPLUS procedure provides more conservative support design forces than traditional M-O approach. If static soil pressures were far from limit conditions, the increases due to the initial Wood effect might have been more significant, as discussed in Becci & Carni (2014). In this case, plastic correction to pseudo-static deformed shape is not possible since no  $\beta$  factor has been computed.

## Conclusions

A simple algorithm to perform pseudo-static analysis of flexible retaining walls with the non-linear spring method has been reviewed. This proposal can be easily included in any design tool offering this approach and can be considered an alternative to traditional approaches based on Wood or to M-O seismic thrusts. It must be clearly remarked that such proposal, as any other pseudo-static approach, is usually not appropriate to predict post-earthquake wall deformations when reduced design acceleration due to ductility allowance is considered in the light of PBDM: just a rough estimate of the expected total wall deformation may be attempted, if the threshold acceleration is computed, as discussed in Example 1. For propped flexible walls PPLUS approach predicts higher internal forces than those given by the M-O assumption, on the safe side. Once such limitations have been clearly understood, such pseudo-static method can be adopted, at least in the early design stages, for a wide range of practical retaining wall scenarios. Very often multi-propped flexible walls act as temporary structures which are not checked under seismic conditions: nevertheless further research including experimental measures would be welcome to improve practical pseudo-static procedures within the Performance Based Design Method and Limit State approaches.

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