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The Use of Wavenumber Normalization in Computing Spatially Averaged Coherencies (krSPAC) of Microtremor Data from Asymmetric Arrays

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ABSTRACT

The spatially-averaged coherency (SPAC) method of processing microtremor noise observations for estimation of Vs profiles requires a circular or triangular array symmetry in order to allow spatial (azimuthal) averaging of inter-station coherencies over a constant station separation. Common processing methods allow for station separations to vary by typically ±10% in the azimuthal averaging before degradation of the SPAC spectrum is excessive. In this paper we develop a new wavenumber-normalised SPAC method (krSPAC). The traditional analysis of SPAC data involves performing averaging of sets of coherency versus frequency spectra and then fitting to a model SPAC spectrum. In our new approach we interpolate each spectrum to coherency versus *kr*, where *k* and *r* are wavenumber and station separation respectively, and *r* may be significantly different for each pair of stations. The averaging and interpolation changes with each iteration of velocity models, since *k* is a function of frequency and phase velocity and thus is updated each iteration. The method proves robust and is compared with alternative methodologies using asymmetric arrays in the Santa Clara Valley CA, Pleasanton CA, and Seattle WA, where station spacings are irregular and vary from 300m to 2000m.

Introduction

Array-based microtremor methods make use of the fact that background seismic noise is dominated by surface waves; if only vertical components are measured, then it is the Rayleigh-wave fraction of energy that is observed. The use of an array allows measurement of the dispersive property of Rayleigh waves (phase velocities vary with frequency) from which the shear-wave velocity Vs profile (variation with depth) may be extracted for the location.

We use the multiple-mode spatially-averaged coherency method (MMSPAC) described by Asten *et al*. (2004) and Asten (2006a). The method builds on the basic spatial autocorrelation (SPAC) method described by Aki (1957) and Okada (2003), but differs from the majority of implementations (such as Picozzi and Albarello, 2007; Di Giulio *et al.*, 2012) by making use of direct fitting of averaged inter-station coherency spectra for layered-earth models (also used by Wathelet, 2005), rather than the more common approach of extracting phase-velocity dispersion curves followed by inversion of the dispersion data for a layered-earth model.

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The method has been applied in urban areas (Roberts and Asten, 2007; Stephenson *et al.*, 2009), suburban areas (Boore and Asten, 2008; Claprood and Asten, 2009; Asten *et al.*, 2013), and remote areas (Schramm *et al.*, 2012; Smith *et al.*, 2013). The direct fitting method has particular advantages in allowing use of higher frequencies than can generally be accessed when using the more common approach of inversion of the phase velocity dispersion curve to a layered-earth model of shear-wave velocities (Asten, 2006b). The algorithm used in the earlier references is augmented here by modelling the "effective Rayleigh mode" which is the summation of multiple modes based on theoretical energy partition between modes for ideal vertical impacts on a layered earth (Arai and Tokimatsu 2004, 2005; Ikeda *et al.*, 2012, 2013).

The majority of published studies using microtremor methods record three-component data on at least one sensor, allowing the horizontal:vertical spectral ratio (HVSR) method to be used as a further constraint on the shear-wave velocity profile. It is generally true that the HVSR method allows use of lower frequencies than does the SPAC method alone (see for example Hayashi *et al.*, 2011; Asten *et al.*, 2014).

The SPAC method depends for its effectiveness on azimuthal averaging of inter-station coherencies either by an azimuthal distribution of wave propagation directions, or use of multiple pairs of stations distributed in azimuth in an array, or both. Thus an array of seismometers arranged in one or more equilateral triangles is a common approach. Asten *et al*. (2014) give examples of how this may be achieved in areas of limited access. Bettig *et al*. (2001) developed an extended methodology called Modified SPAC (MSPAC) allowing azimuthal averaging using arrays containing some departures from symmetry, a method implemented in the public domain geopsy software (http://www.geopsy.org/). In this study we extend the direct-fitting MMSPAC method to allow its use with arbitrarily asymmetric arrays.

Azimuthal Averaging with Irregular Station Spacing using krSPAC

The concept of azimuthal averaging of coherencies for a plane wave passing multiple pairs of stations can be expressed, following Aki (1957), Okada (2003) and Asten (2006a), in the form

$$\rho(f) = \int e^{\,i\,k.r\cos\theta}\, d\,\theta = Jo\,(kr), \qquad (1)$$

where ρ is the coherency spectrum, f is frequency, $k = 2\,\pi\, f \,/\, C(f)$ is the wavenumber, $C(f)$ is the phase velocity dispersion curve, r is a constant station separation, θ is azimuthal angle of a station pair relative to the plane wave vector, and Jo is the Bessel function of zero order. When performing interpretation via direct fitting of SPAC spectra, we use Equation 1 with a known value of r and a forward model dispersion curve $C(f)$ to compute a model SPAC spectra which is fitted to the observed azimuthally averaged SPAC spectrum (e.g., Figure 1).The direct fitting is achieved by iterative forward modelling; the quality of the fit over a given band-width is measured objectively by the standard deviation of the fit, and variables in a layered earth model (shear-wave velocity *Vs*, and thickness *h*) are varied to achieve a best fit via a least-squares criterion. Further details and examples are given in Asten et al (2014).

For an array of four stations in a centred equilateral triangle (such as triangle ABC of Figure 2), the azimuthally averaged observed SPAC spectrum becomes

$$\rho(f) = [\ \rho_1(f) + \rho_2(f) + \rho_3(f)]\ /3 \qquad (2)$$

where the ρ_i are coherency spectra for individual pairs of stations. If the wave field is omnidirectional, each of the three terms in Equation 3 approximates a Bessel function and Equation 2 may be written

$$\rho(kr) = [\ \rho_1(kr_1) + \rho_2(kr_2) + \rho_3(kr_3)]\ /3 \qquad (3)$$

$$\approx [\ Jo(kr_1) + Jo(kr_2) + Jo(kr_3)]\ /3 \qquad (4)$$

where the r_i are equal and can be either the radius or the side-length of the triangle. If the triangle is slightly irregular in shape then the three *Jo* terms are no longer additive at higher wavenumbers and the shape of the averaged coherency curve cannot be solved for wavenumber and phase velocity. Figure 1 shows an example where the r_i differ by 15% (a common criterion when using the MSPAC approach).

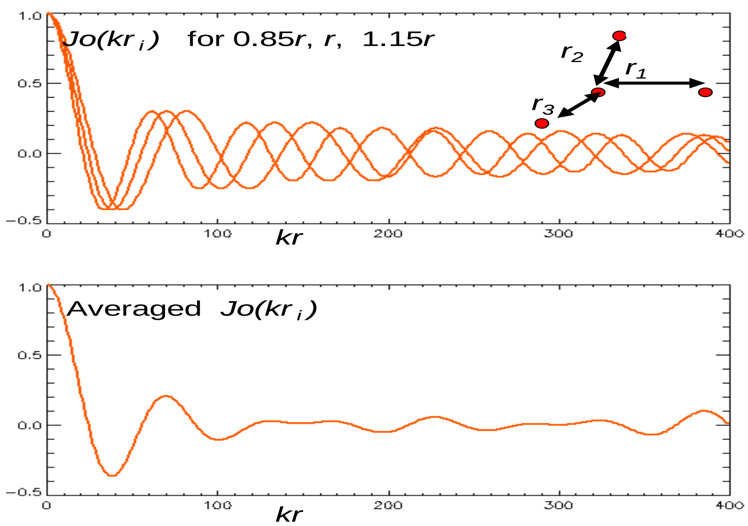

Figure 1. Addition of three ideal inter-station coherency spectra where station separations vary by 15%. In this example the averaged spectrum is useful only to the 2nd minimum.

For an assumed dispersion curve *C(f)* and a single average r-value for an array we can use Equation 3 to sum observed SPAC spectra for multiple pairs of stations (the MMSPAC approach). For an asymmetric array the known r-values the r_i are not equal but Equation 3 remains valid as a method to sum observed SPAC spectra. As a detail of implementation the abscissa for the three complex spectra represented in Equation 3 have different sampling due to the different values of r_i, and it is necessary to resample the spectra using a cubic spline in order to perform the addition. The averaged observed spectrum $\rho(kr)$ changes with each iteration involving an updated model dispersion, but this presents no problem in implementing the iterative inversion process previously described. We have given the method the name krSPAC as described in Asten *et al*. (2013).

The method is tested on data from the Saratoga test site, San Jose California, where borehole shear-wave velocity (Vs) logs to 300m depth are available, and a range of active and passive seismic methods have been applied (see Figure 17 of Boore and Asten, 2008). Figure 2 shows the layout of a pair of triangular arrays and an asymmetric triangle used to test the krSPAC method.

Figure 3 shows an example of MMSPAC interpretation using the large equilateral triangle of Figure 2 (as reported in Boore and Asten, 2008). Figures 4 and 5 show the equivalent plots using krSPAC. It is evident that the krSPAC averaging preserves information in the higher-order maxima and minima of the *Jo* function, even though the inter-station distances differ by up to 30% from the mean.

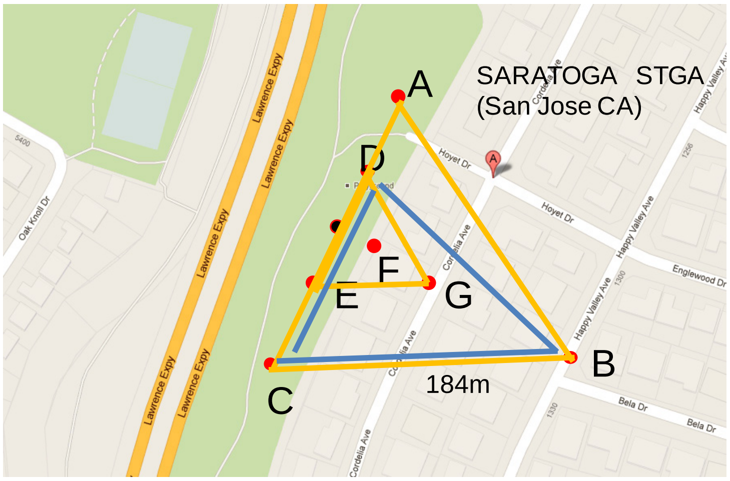

Figure 2. Seismic arrays used for microtremor recordings at Saratoga (STGA) site. Yellow lines: equilateral triangles used for MMSPAC analysis. Blue lines: an asymmetric array used for krSPAC analysis.

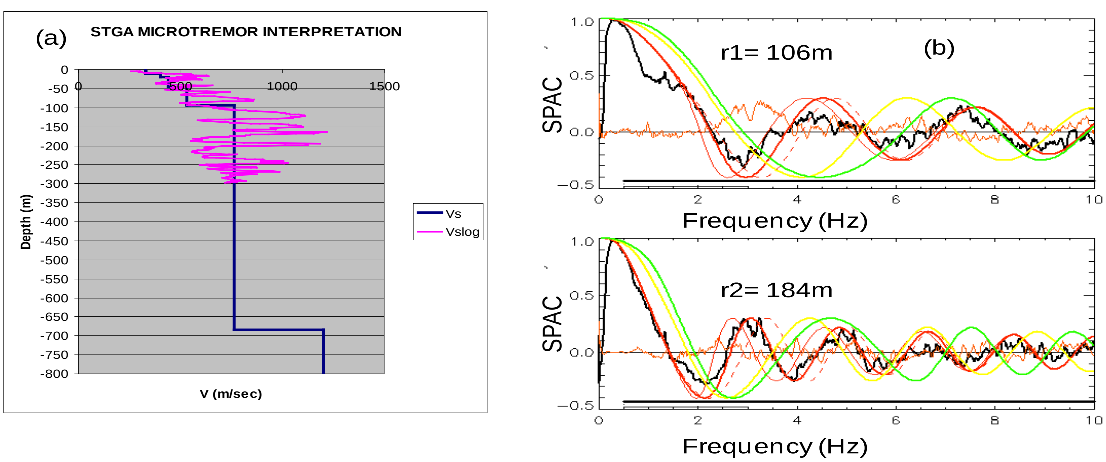

Figure 3. (a) Best-fit layered-earth model for STGA site, with (pink line) a borehole Vs log. (b) MMSPAC interpretation using fitted field and model SPAC spectra for large equilateral triangle array ABC of Fig. 2. Black: observed MMSPAC. Thick red: best-fitmodel SPAC for fundamental Rayleigh mode using the layered-earth model from (a). Yellow, green lines: model curves for 1^{st} and 2^{nd} higher modes. Thin red line and dashed line: sensitivity plot for layer 6 (50-100m depth) having Vs varied ± 10%.

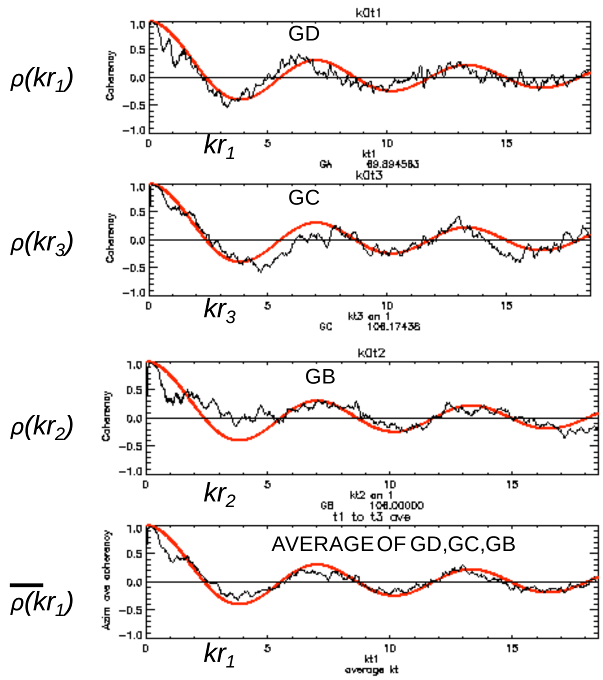

Figure 4. Summation using Equation 3 for krSPAC. Black lines: Inter-station coherency spectra for asymmetric triangle, station pairs GD, GC, GB, and the krSPAC average. Red line: Theoretical *Jo*(kr) using the same layered-earth model dispersion curve and the varying r-values.

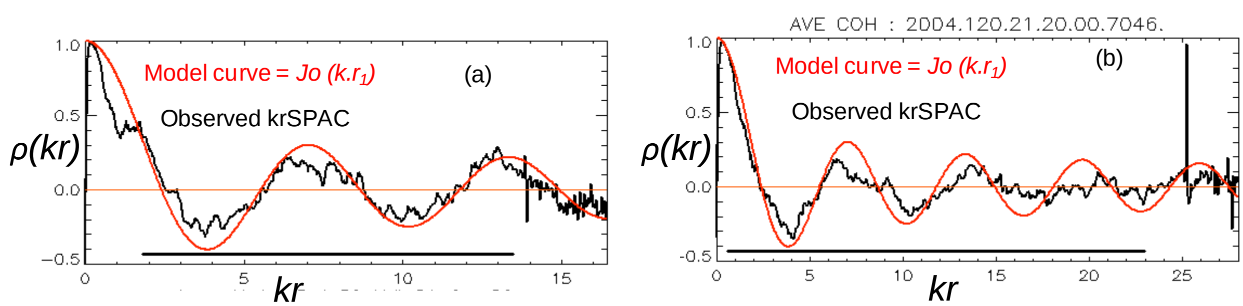

Figure 5. Black: krSPAC observed spectra for (a) radii and (b) circumference of the STGA asymmetric triangle array DBC. Red: best-fit model (same model as in Fig. 3).

krSPAC Applied to the Pleasanton 1km diameter array

Ambient noise recorded on an asymmetric seismic array installed in suburban Pleasanton, California (Figure 6a) is used in a direct comparison of interpretation by MSPAC (Figure 6b) and krSPAC (Figure 6c). With MSPAC the maximum useful frequency was about 1 Hz, and the interpretation yielded a three-layer model with the top layer 150m thick and undifferentiated (Figure 7). The useful upper frequency with krSPAC is 5x larger than for MSPAC, and the corresponding useful Rayleigh wavelengths are about 15x shorter, giving resolution of Vs for the upper 7 m of soil. Averaged Vs for the upper 30m and upper 100m are given in Table 1; it is evident that the data accessible from higher frequencies utilized by krSPAC have a major influence on estimation of soil conditions, and hence earthquake hazard, in the upper 100m.

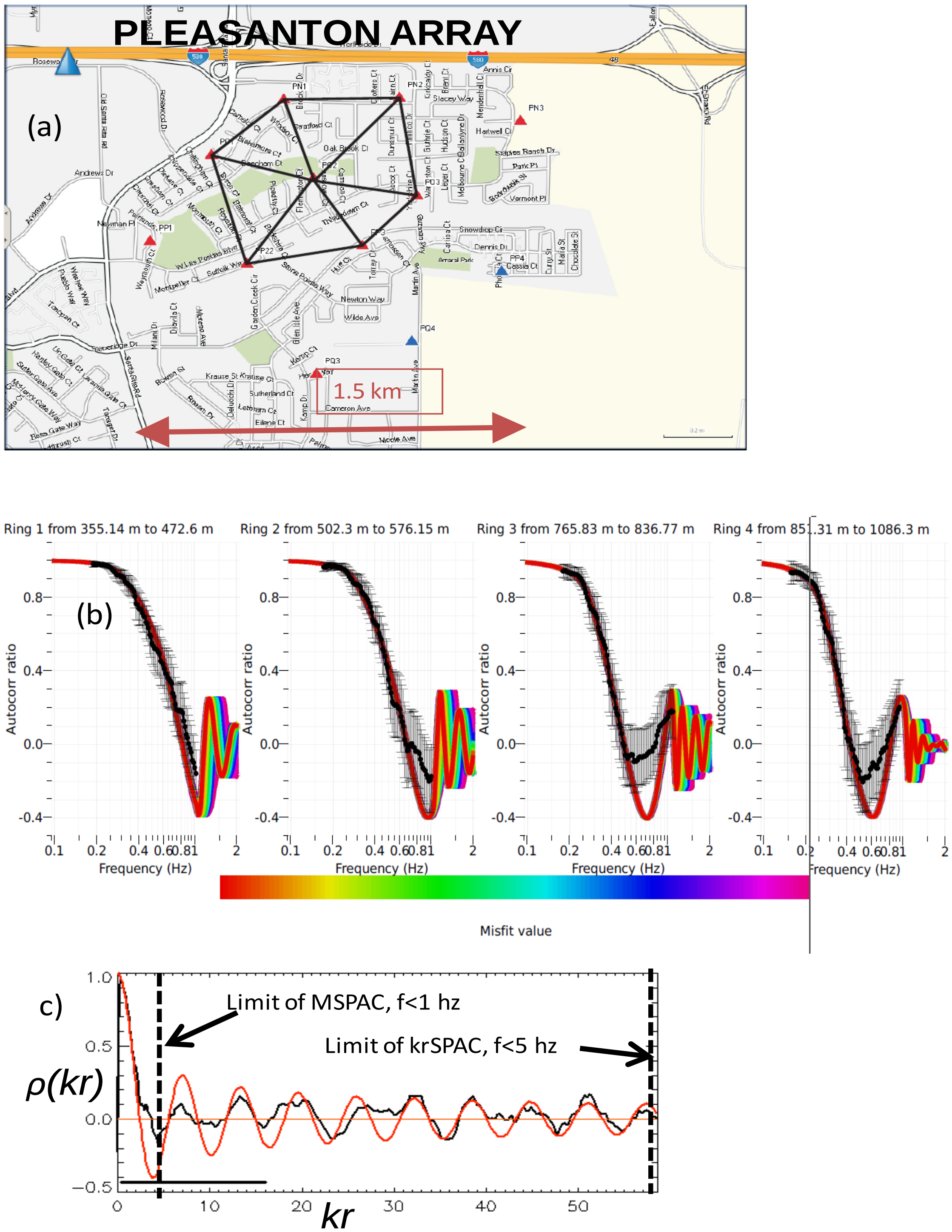

Figure 6. (a) Pleasanton seismic array, California. (b) MSPAC interpretation on seven-station array, using geopsy software; useful frequencies are below 1 Hz with average station spacings 410m, 530m, 800m, 960m. (c) krSPAC fit on circumference of western triangle sub-array (r ≈ 520m), using frequencies up to 5 Hz.

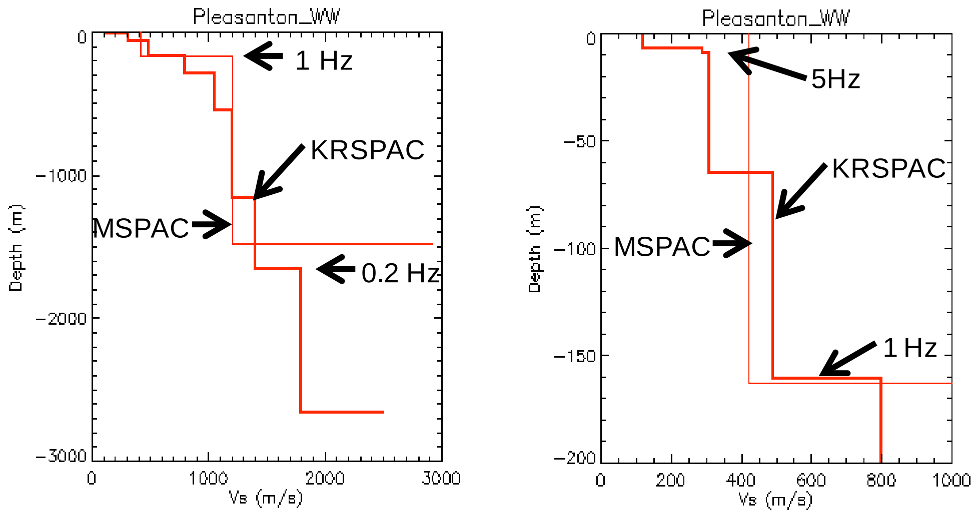

Figure 7. Vs versus depth interpretations of Pleasanton data. Thick red line: krSPAC on western triangle. Thin red line: MSPAC on 7-station array. Arrowed interfaces are resolved by frequency bands approximately centred on frequencies marked.

Table 1. Average velocity Vs for upper layers for Pleasanton, using the best-fit layered-earth model from each of two processing methods.

Ave Vs	MSPAC	krSPAC
Vs30	422 m/s	226 m/s
Vs100	599 m/s	315 m/s

krSPAC in the Seattle Basin – Comparison with Vs Tomography

Delorey and Vidale (2011) used an irregular grid of 87 stations over the Seattle Basin, processing data at frequencies below 0.5 Hz with ambient noise Rayleigh-wave tomography to develop a 3D Vs model. The terrain makes placement of regularly spaced arrays for SPAC methodology difficult, however the krSPAC method allows the use of irregular arrays placed on available access roads. We show an example for a single site in Figure 8, where the Vs profile from Delorey and Vidale (2011) is compared with a Vs profile obtained from microtremor observations using three nested asymmetric triangles. The tomographic measurement does not give information in the upper 250m, whereas the krSPAC data has resolved Vs in the upper 20m. The Vs100 values for tomography and krSPAC models are respectively 740m/s and 518m/s. As with comparisons between MSPAC and krSPAC in the previous section, the difference is significant for earthquake hazard calculations, hence we conclude that the krSPAC approach provides valuable additional information. The tomographic data were acquired over a five-month time span, whereas the krSPAC data (three triangles) were prepared, acquired and removed in 6 hours.

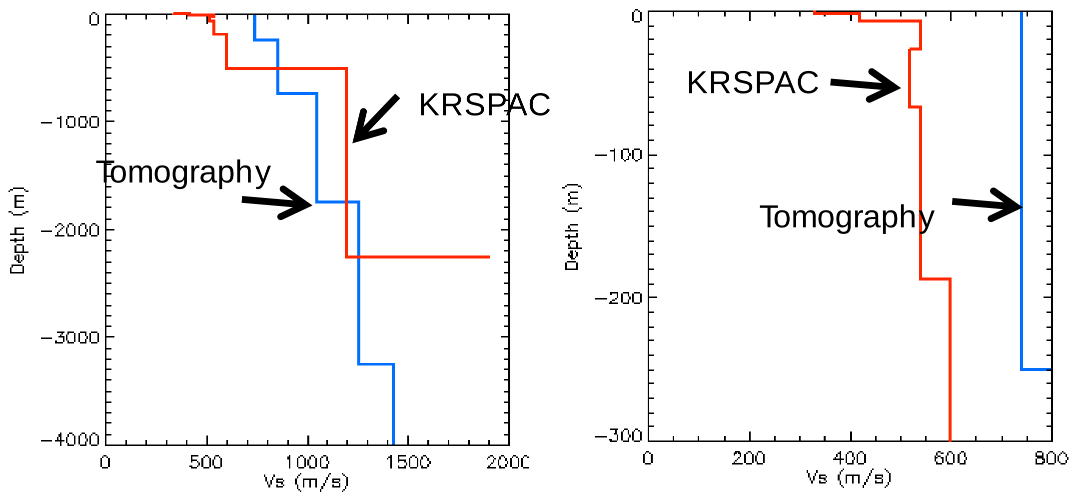

Figure 8. Site 3, Seattle Basin (122°21.4' W, 47°42.7' N). Red: Vs profile from krSPAC data using three irregular triangles, side-lengths averaging 300, 1000, 2000m, Rayleigh wave frequencies 0.2 to 8 Hz. Blue: Vs profile from Delorey and Vidale (2011) which used Rayleigh wave tomography, frequencies 0.1 to 0.5 Hz. The right panel shows Vs in the upper 300m, illustrating how krSPAC has provided additional near-surface resolution relative to tomography.

Conclusions

The krSPAC method allows inversion of microtremor Rayleigh-wave data from asymmetric arrays, over a wider frequency band than alternative methods, with *kr* (unitless) values up to 25 and 60 achieved in examples described here. Thus despite the large and irregular station spacings ranging from 100 to 2000m, this method permits resolution of Vs within the upper 30m of near-surface sediments, and down to a maximum depth of 2.5km.

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